Week #1
Week #2

Domain #1: The Heart of Algebra

Weekly Example Problems

Example #1: In 2014, County X had 783 miles of paved roads. Starting in 2015, the country has been building 8 miles of new paved roads each year. At this rate, how many miles of paved road will Country X have in 2030? (Assume that no paved roads go out of service).

Note that this question has no choices. It is a student-produced response question. On the SAT, you would grid your answer in the spaces provided on the answer sheet.

How does the number of miles depend on the year?
Let n = number of years after 2014 (since that's the year the question started with). Each year there are 8 more miles of paved road → 8n
But we started with 783 miles $8n+783$
2030 is 16 years after 2014
$8(16)+783 = 911$ miles

Example #2: In 2014, County X had 783 miles of paved roads. Starting ~~in 2015~~, the country has been building 8 miles of new paved roads each year. At this rate, if *n* is the number of years of 2014, which of the following functions *f* gives the number of miles of paved road there will be in Country X? (Assume that no paved roads go out of service.)

A) $f(n) = 8 + 783n$
B) $f(n) = 2014 + 783n$
C) $f(n) = 783 + 8n$
D) $f(n) = 2014 + 8n$

This question is easier than Ex 1 because it already defines the variable & you don't have to solve for anything.

Example #3: In 2014, County X had 783 miles of paved roads. Starting in 2015, the country has been building 8 miles of new paved roads each year. At this rate, in which year will Country X first have at least 1,000 miles of paved roads? (Assume that no paved roads go out of service.)

y=1000

Let y = miles of paved roads in year x
Let n = years after 2014

at least 1000 miles

$8n+783 \geq 1000$
$8n \geq 217$
$n \geq 27.125$

→ we are asked what year it will be, so 28 years after 2014 = 2042

Example #4: To edit a manuscript, Miguel charges \$50 for the first 2 hours and \$20 per hour after the first 2 hours. Which of the following expresses the amount, C, in dollars, Miguel charges if it takes him x hours to edit a manuscript, where $x > 2$?

1 hour = \$50
~~2 hours = \$50~~
3 hours = \$50 + \$20 = \$70
4 hours = \$50 + \$40 = \$90

A) $C = 20x$
B) $C = 20x + 10$
C) $C = 20x + 50$
D) $C = 20x + 90$

He doesn't charge \$20/hr until after the first 2 hours so C is wrong. D means \$20 an hr plus a flat fee of \$90. A means just \$20/hr (not charging for the first 2 hours).

Example #5: A builder uses the function g defined by $g(x) = 80x + 10,000$ to estimate the cost $g(x)$ in dollars to build a one-story home of planned floor area of x square feet. If the builder estimates that the cost to build a certain one-story home is \$106,000, what is the planned floor area, in square feet, of the home?

$g(x)$

$g(x) = 80x + 10,000$

$106,000 = 80x + 10,000$
$-10,000 \qquad -10,000$

$\frac{96,000}{80} = \frac{80x}{80}$

$x = 1200$ sq. ft

Example #6: Maizah bought a pair of pants and a briefcase at a department store. The sum of the ~~process~~ prices of the pants and the briefcase before sales tax was \$130. There was no sales tax on the pants and a 9% sales tax on the briefcase. The total Maizah paid, including sales tax, was #136.75. What was the price, in dollars, of the pants?

Let p = pants
b = briefcase
$130 - 75 = \$55$

$p + b = 130$
$1.09b + p = 136.75$
$1.09b + 130 - b = 136.75$
$.09b = 6.75 \quad b = 75$

The briefcase had 9% tax
$9\% = .09$
so briefcase cost is
$b + .09b = 1.09b$

Example #7: Each morning, John jogs 6 miles per hour and rides a bike at 12 miles per hour. His goal is to jog and ride his bike a total of at least 9 miles in no more than 1 hour. If John jogs j miles and rides his bike b miles, which of the following systems of inequalities represents John's goal?

A) $\frac{j}{6} + \frac{b}{12} \leq 1$
$j + b \geq 9$

~~B)~~ $\frac{j}{6} + \frac{b}{12} \geq 1$
$j + b \leq 9$

~~C)~~ $6j + 12b \geq 9$
$j + b \leq 1$

D) $6j + 12b \leq 1$
$j + b \geq 9$

j = miles jogged
b = miles biked

$j + b \geq 9$

distance = rate x time

time = $\frac{\text{distance}}{\text{rate}}$

so $\frac{j}{6} + \frac{b}{12} \leq 1$ hour

Example #8: What is the solution to the equation: $3\left(\frac{1}{2} - y\right) = \frac{3}{5} + 15y$

$\frac{3}{2} - 3y = \frac{3}{5} + 15y$

$\frac{3}{2} - \frac{3}{5} = 18y$

$\frac{15}{10} - \frac{6}{10} = 18y$

$\frac{\frac{9}{10}}{18} = \frac{18y}{18}$

$\frac{9}{10} \cdot \frac{1}{18} = y$

$\frac{1}{20} = y$

Example #9: What is the solution to the equation: $-2(3x - 2.4) = -3(3x - 2.4)$

$-6x + 4.8 = -9x + 7.2$

$+9x \; -4.8 \quad +9x \; -4.8$

$\frac{3x}{3} = \frac{2.4}{3}$ → $x = .8$

Example #10: What is the solution (x, y) to the system of equations:

$-2x = 4y + 6$

$2(2y + 3) = 3x - 5$

$\frac{-2x}{-2} = \frac{4y+6}{-2}$

$x = -2y - 3$

$2(2y+3) = 3(-2y-3) - 5$

$4y + 6 = -6y - 9 - 5$

$10y = -20$

$y = -2$

$x = -2(-2) - 3$

$x = 4 - 3$

$x = 1$

$\boxed{(1, -2)}$

Example #11: How many solutions (x, y) are there to the system of equations:

$2y + 6x = 3$

$-2(y + 3x = 2)$

$2y + 6x = 3$

$-2y - 6x = -4$

$0 = -1$

False

no solution

A) Zero
B) One
C) Two
D) More than two

x or y = # → one solution

= # that's true ∞ solutions

= # that's false no solution

* if you graph them they would be parallel → never cross = no solution

Example #12: In the system of equations below, *a* and *b* are constants. If the system has infinitely many solutions, what is the value of *a*?

For this to happen, both equations have to be equal

$-5(3s - 2t = a)$

$-15s + bt = -7$ → $b = 10$

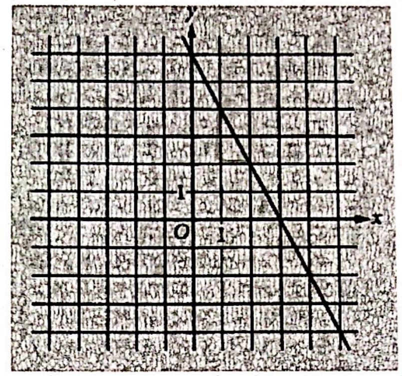
$-15s + 10t = -5a$

$-15s + bt = -7$

and $-5a = -7$

$a = \frac{7}{5}$

Example #13:



Slope is $\frac{\text{rise}}{\text{run}}$

$\frac{-2}{1} = -2$

The graph of line *k* is shown in the *xy*-plane above. Which of the following is an equation of a line that is perpendicular to line *k*?

A) $y = -2x + 1$
B) $y = -\frac{1}{2}x + 2$
C) $y = \frac{1}{2}x + 3$
D) $y = 2x + 4$

$y = mx + b$
$m = \text{slope}$

perpendicular lines have opposite, reciprocal slopes
The slope of the line in the graph is -2
opposite reciprocal is $\frac{1}{2}$

Example #14: A voter registration drive was held in Town Y. The number of ~~votes~~ voters, *V*, registered *T* days after the drive began can be estimated by the equation $V = 3450 + 65T$. What is the best interpretation of the number 65 in this equation?

A) The number of registered voters at the beginning of the registration drive
B) The number of registered votes at the end of the registration drive
C) The total number of voters registered during the drive
D) The number of voters registered each day during the drive

$V = 3450 + 65T$

days

number of voters registered

65 is voters registered per day

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