Calculus
Trig. Derivatives
Worksheet #3

Name: Key

Directions: Find the derivative of the following functions.

1. $f(x) = [\sec(-x)][\sin(3x^2 - 7x)]$

$f'(x) = [\sec(-x)][\sin(3x^2-7x)]' + [\sin(3x^2-7x)][\sec(-x)]'$

$f'(x) = [\sec(-x)][\cos(3x^2-7x)][6x-7] + [\sin(3x^2-7x)][\sec(-x)\tan(-x)(-1)]$

2. $f(x) = \cot\left[\frac{(\ln x)}{e^{-x}}\right]$

$f'(x) = -\csc^2\left[\frac{\ln x}{e^{-x}}\right]\left[\frac{\ln x}{e^{-x}}\right]'$

$f'(x) = -\csc^2\left[\frac{\ln x}{e^{-x}}\right]\left[\frac{(e^{-x})(\frac{1}{x}) - (\ln x)(e^{-x})(-1)}{(e^{-x})^2}\right]$

3. $f(x) = \frac{\cos(\sqrt{5x^2-3})}{\tan x} = \frac{\cos(5x^2-3)^{1/2}}{\tan x}$

$f'(x) = \frac{(\tan x)(\cos(5x^2-3)^{1/2})' - [\cos(5x^2-3)^{1/2}](\tan x)'}{(\tan x)^2}$

$f'(x) = \frac{[(\tan x)(-\sin(5x^2-3)^{1/2})(\frac{1}{2}(5x^2-3)^{-1/2})(10x)] - [\cos(5x^2-3)^{1/2}][\sec^2 x]}{(\tan x)^2}$

4. $f(x) = \csc^2(8x) - \cot^2(8x)$

$f(x) = (\csc(8x))^2 - (\cot(8x))^2$

$f'(x) = 2(\csc(8x))(-\csc(8x)\cot(8x)(8)) - 2(\cot(8x))(-\csc^2(8x)(8))$

$\boxed{\underline{\text{OR}} \text{ since } 1 + \cot^2 x = \csc^2 x \rightarrow \csc^2 x - \cot^2 x = 1 \text{ so } \frac{d}{dx} \text{ is } 0}$

5. $f(x) = \sec(5x\sin(5x))$

$f'(x) = [\sec(5x\sin(5x))\tan(5x\sin(5x))][(5x)(\cos(5x))(5) + (\sin(5x))(5)]$

6. $f(x) = \sin(\cos(3\pi x))$

$f'(x) = [\cos(\cos(3\pi x))][-\sin(3\pi x))(3\pi)]$

7. $f(x) = \sqrt[3]{\csc\sqrt{2\sin x\cos x}} = \left(\csc(2\sin x\cos x)^{1/2}\right)^{1/3}$

or rewrite $2\sin x\cos x$ as $\sin(2x)$

$$f'(x) = \frac{1}{3}\left(\csc(2\sin x\cos x)^{1/2}\right)^{-2/3}\left(-\csc(2\sin x\cos x)^{1/2}\cot(2\sin x\cos x)^{1/2}\right)(\sin x\cos x)^{-1/2}\left[(2\sin x)(-\sin x) + (\cos x)(2\cos x)\right]$$

8. $f(x) = \ln 5^{x(\sec 3x)(\cos -2x)} = [(x)(\sec 3x)](\cos(-2x))\ln 5$

$$f'(x) = \ln 5\left[\left((x)(\sec(3x))\right)(-\sin(-2x)(-2)) + (\cos(-2x))\left[(x)(\sec(3x)\tan(3x)(3)) + (\sec(3x)(1))\right]\right]$$

9. $f(x) = \log[(\cos^2 x + 2\tan(x^3 - 7x)] = \log[(\cos x)^2 + 2\tan(x^3 - 7x)]$

$$f'(x) = \frac{1}{\ln 10}\left[\frac{2(\cos x)(-\sin x) + 2[\sec^2(x^3-7x)(3x^2-7)]}{(\cos x)^3 + 2\tan(x^3-7x)}\right]$$

10. $f(x) = [\csc(e^{-x})][\ln(\sin 3x)]$

$$f'(x) = \left[(\csc(e^{-x}))\left(\frac{(\cos(3x))(3)}{\sin 3x}\right)\right] + \left[(\ln(\sin(3x)))(-\csc(e^{-x}))(\cot(e^{-x}))(-1)\right]$$

11. $f(x) = \sec^4(\sin(\tan\sqrt{x})) = [\sec(\sin(\tan x^{1/2}))]^4$

$$f'(x) = 4[\sec(\sin(\tan x^{1/2}))]^3\left[(\sec(\sin(\tan x^{1/2})))(\tan(\sin(\tan x^{1/2})))(\cos(\tan x^{1/2}))(\sec^2(x^{1/2})(\tfrac{1}{2}x^{-1/2}))\right]$$

12. $f(x) = \frac{2}{\sqrt[5]{\ln[3e^x(\cot x)]}} = 2[\ln 3 + x + \ln(\cot x)]^{-1/5}$

$$f'(x) = -\frac{2}{5}[\ln 3 + x + \ln(\cot x)]^{-6/5}\left[0 + 1 + \left(\frac{-\csc^2 x}{\cot x}\right)\right]$$

13. $f(x) = (x^{\cos(-\pi)})(-\pi\cos x) = (x^{-1})(-\pi\cos x)$

$$f'(x) = (x^{-1})(-\pi\cos x)' + (-\pi\cos x)(x^{-1})'$$
$$= (x^{-1})(-\pi(-\sin x)) + (-\pi\cos x)(-x^{-2})$$

14. $f(x) = \frac{\frac{\cos(5x)}{\cos(-3x)}}{\csc(-3x)\cot(5x)}$

OR simplify

$$= \frac{\frac{\cos(5x)}{\cos(-3x)}}{\left(\frac{1}{\sin(-3x)}\right)\left(\frac{\cos(5x)}{\sin(5x)}\right)} = \frac{\cos(5x)}{\cos(-3x)}\cdot\frac{\sin(-3x)\sin(5x)}{\cos(5x)} = \tan(-3x)\sin(5x)$$

$$f'(x) = [(\tan(-3x))(\cos(5x))(5)] + [(\sin(5x))(\sec^2(-3x))(-3)]$$

Long way

$$f'(x) = \frac{(\csc(-3x)\cot(5x))\left(\frac{(\cos(-3x))(-\sin(5x))(5)) - (\cos(5x))(-\sin(-3x))(-3))}{(\cos(-3x))^2}\right) - \left(\frac{\cos(5x)}{\cos(-3x)}\right)\left[((\csc(-3x))(-\csc^2(5x))(5)) + ((\cot(5x))(-\csc(-3x)\cot(-3x)(-3))\right]}{(\csc(-3x)\cot(5x))^2}$$