KEY

Trigonometric Derivatives & the Chain Rule

More Trig. Derviative Rules! 

1. $\frac{d}{dx}[\sin f(x)] = (\cos f(x))(f'(x))$

2. $\frac{d}{dx}[\cos f(x)] = (-\sin f(x))(f'(x))$

3. $\frac{d}{dx}[\tan f(x)] =$ $(\sec^2 f(x))(f'(x))$

4. $\frac{d}{dx}[\sec f(x)] =$ $(\sec f(x))(\tan f(x))(f'(x))$

5. $\frac{d}{dx}[\csc f(x)] =$ $(-\csc f(x))(\cot f(x))(f'(x))$

6. $\frac{d}{dx}[\cot f(x)] =$ $(-\csc^2 f(x))(f'(x))$

Examples: Find the derivative of the following functions.

1. $y = \sin(x^2 - 3x - 5)$ $f(x) = x^2 - 3x - 5$ $f'(x) = 2x - 3$

$y = (\cos(x^2 - 3x - 5))(2x - 3)$

2. $y = \sin^4 3x - \cos^4 3x$

$y = (\sin(3x))^4 - (\cos(3x))^4$

$y' = 4(\sin(3x))^3(\cos(3x))(3) - 4(\cos(3x))^3(-\sin(3x))(3)$

3. $y = 2\cot(\ln x^4 + 3e^{-2x})$ $f(x) = 4\ln x + 3e^{-2x}$ $f'(x) = \frac{4}{x} + (3e^{-2x})(-2)$

$y = 2\cot(4\ln x + 3e^{-2x})$

$y = 2(-\csc^2(4\ln x + 3e^{-2x}))\left(\frac{4}{x} + (3e^{-2x})(-2)\right)$

4. $y = 3\sec\left[\ln\left(5x^3 - \frac{2}{x}\right)\right]$

$f(x) = \ln\left(5x^3 - \frac{2}{x}\right)$

$f'(x) = \frac{15x^2 + 2x^{-2}}{5x^3 - \frac{2}{x}}$

$y = 3\sec\left[\ln\left(5x^3 - \frac{2}{x}\right)\right]\tan\left[\ln\left(5x^3 - \frac{2}{x}\right)\right]\left[\frac{15x^2 + 2x^{-2}}{5x^3 - \frac{2}{x}}\right]$

5. $y = \csc\left[\frac{(\tan e^{3x})^2}{\cos x}\right]$

$f(x) = \left[\frac{(\tan e^{3x})^2}{\cos x}\right]$

$y' = -\csc\left[\frac{(\tan e^{3x})^2}{\cos x}\right]\cot\left[\frac{(\tan e^{3x})^2}{\cos x}\right]$ times

$f'(x) = \frac{(\cos x)\left[(\tan e^{3x})^2\right]' - (\tan e^{3x})^2(\cos x)'}{\cos^2 x}$

$f'(x) = \frac{(\cos x)\left(2(\tan e^{3x})(\sec e^{3x})(e^{3x})(3)\right) - (\tan e^{3x})^2(-\sin x)}{\cos^2 x}$

6. $y = \ln[\tan(\cos(2^{-x}))]$

$y' = \frac{\sec^2(\cos(2^{-x}))(-\sin(2^{-x}))(\ln 2)(2^{-x})(-1)}{\tan(\cos(2^{-x}))}$

$f(x) = \cos(2^{-x})$

$f'(x) = -\sin(2^{-x})(\ln 2)(2^{-x})(-1)$

7. $y = \frac{\cot(3xe^x)}{\cos(-x)} = \frac{\cot(3xe^x)}{\cos x}$

$\frac{(\cos x)(\cot(3xe^x))' - (\cot(3xe^x))(\cos x)'}{\cos^2 x} = \frac{(\cos x)(-\csc^2(3xe^x))(3xe^x + 3e^x) - (\cot(3xe^x))(-\sin x)}{\cos^2 x}$