

# Pre-Calculus Final Review

①  $\theta = \frac{\pi}{6}, \frac{11\pi}{6}$

②  $\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$

③  $\cot \theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{7\pi}{4}$

④  $\sec \theta = -2$

$\cos \theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

⑤  $2 \sin x = \sqrt{2}$

$\sin x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

⑥  $\cos^2 x = \frac{1}{4}$

$\cos x = \pm \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

⑦  $2 \cos x - \sin x \cos x = 0$

$\cos x (2 - \sin x) = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$2 - \sin x = 0$

$2 = \sin x$

NO solution

⑧  $8 \cos x - 4\sqrt{2} = 0$

$8 \cos x = 4\sqrt{2}$

$\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{7\pi}{4}$

⑨  $3 \tan^2 x - 4 = 0$

$3 \tan^2 x = 4$

$\tan^2 x = 1$

$\tan x = \pm 1$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

⑩  $3 \sec^2 x - 4 = 0$

$3 \sec^2 x = 4$

$\sec^2 x = \frac{4}{3}$

$\sec x = \pm \frac{2}{\sqrt{3}}$

$\cos x = \pm \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

⑪  $\sin^2 x - 3 \sin x + 2 = 0$

$(\sin x - 2)(\sin x - 1) = 0$

$\sin x - 2 = 0$

$\sin x = 2$

NO solution

$\sin x - 1 = 0$

$\sin x = 1$

$x = \frac{\pi}{2}$

$$(12) \sin^2 x \cos x - \cos x$$

$$\sin^2 x \cos x - \cos x = 0$$

$$\cos x (\sin^2 x - 1) = 0$$

$$\cos x = 0 \quad \sin^2 x = 1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin^2 x = 1$$

$$\sin x = \pm 1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$(13) \sin x - 2\sin x \cos x = 0$$

$$\sin x (1 - 2\cos x) = 0$$

$$\sin x = 0 \quad 1 - 2\cos x = 0$$

$$x = 0, \pi$$

$$1 = 2\cos x$$

$$\frac{1}{2} = \cos x$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$(14) \log_3 \frac{1}{3} = x$$

$$3^x = \frac{1}{3}$$

$$3^x = 3^{-1}$$

$$x = -1$$

$$(15) \frac{1}{2} \ln e^{10}$$

$$6 \ln e$$

$$6(1) = 6$$

$$(16) \log 1 = 0$$

$$(17) \log (10)^{-25} = -25 \log (10) = -25(1)$$

$$= -25$$

$$(18) \log_4 5 + \log_4 3 - \log_4 60$$

$$\log_4 15 - \log_4 60$$

$$\log_4 \frac{15}{60}$$

$$\log_4 \frac{1}{4} = \log_4 4^{-1} = -1 \log_4 4 = -1$$

$$(19) \ln \sqrt[3]{e^5} + \ln \sqrt{e}$$

$$\ln e^{\frac{5}{3}} + \ln e^{\frac{1}{2}}$$

$$\frac{5}{3}(\ln e) + \frac{1}{2}(\ln e) = \frac{5}{3} + \frac{1}{2} = \frac{10}{6} + \frac{3}{6} = \frac{13}{6}$$

$$(20) \log 500 - \log 50 = \log \frac{500}{50} = \log 10 = 1$$

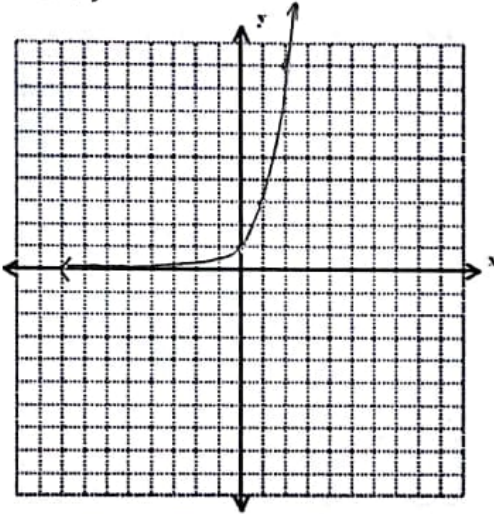
$$(21) \ln e^5 + \log_3 27 - \frac{1}{3} \log_2 8$$

$$5 \ln e + \log_3 3^3 - \frac{1}{3} \log_2 2^3$$

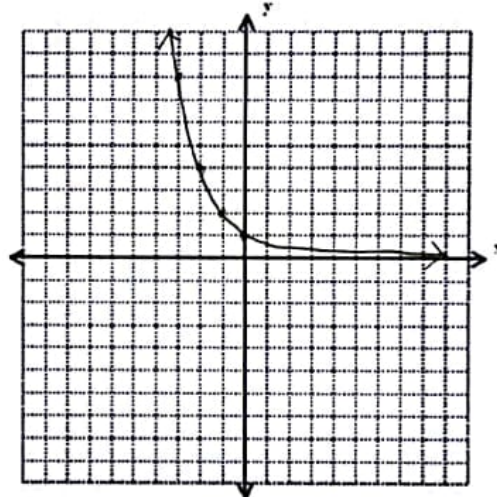
$$5 + 3 - 1 = 7$$

Graph each of the following.

22.  $y = 3^x$



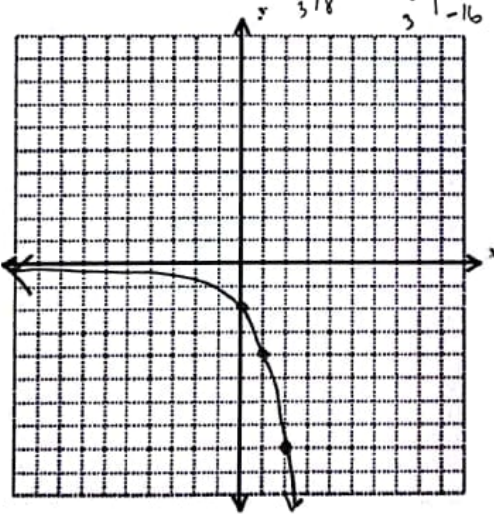
23.  $y = \frac{1}{2}^x$



x-axis flip  
v.s by 2

24.  $y = -2(2)^x$

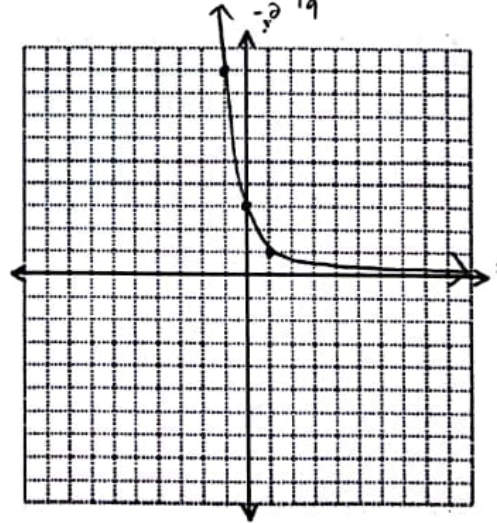
$y=2^x$		$x$	$y$
0	1	0	-2
1	2	1	-4
2	4	2	-8
3	8	3	-16



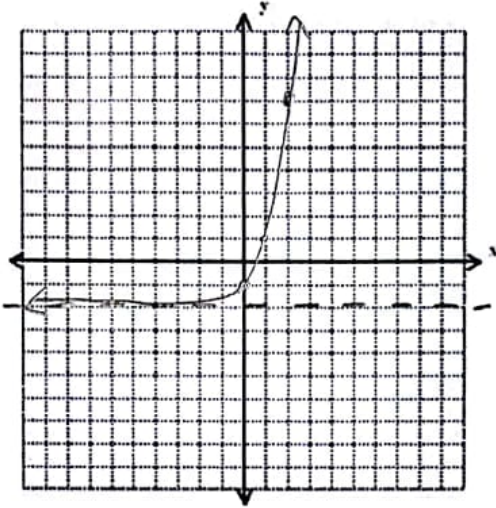
25.  $y = \frac{1}{3}^{x-1}$

$y=3^x$		$x$	$y$
0	1	0	1
1	3	1	3
2	9	2	9

right 1



26.  $y = 3^x - 2$  down 2



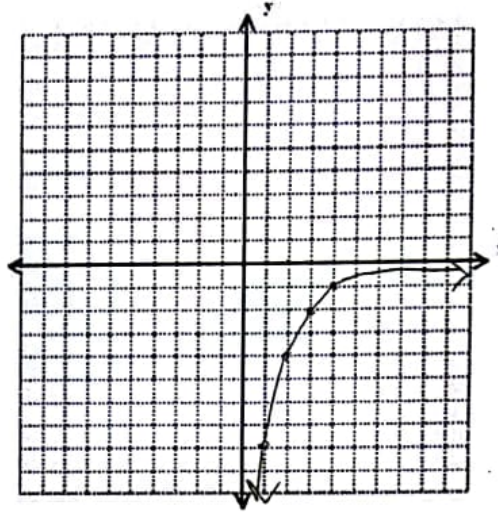
x-axis flip  
right 4

$$y = \frac{1}{2}^x$$

27.  $y = -\frac{1}{2}^{x-4}$

x	y
0	1
-1	2
-2	4
-3	8

x	y
4	-1
3	-2
2	-4
1	-8

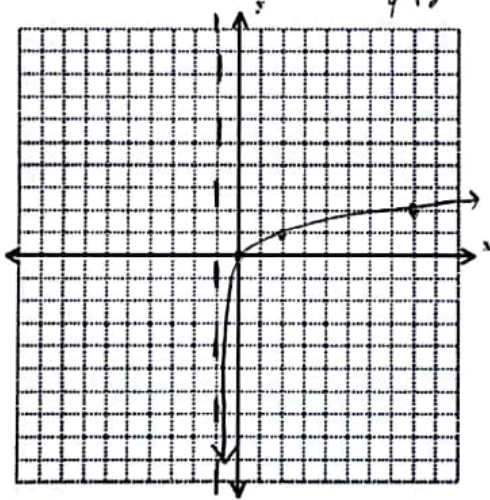


left 1

28.  $y = \log_3(x+1)$

$$y = \log_3 x$$

x	y
1	0
3	1
9	2

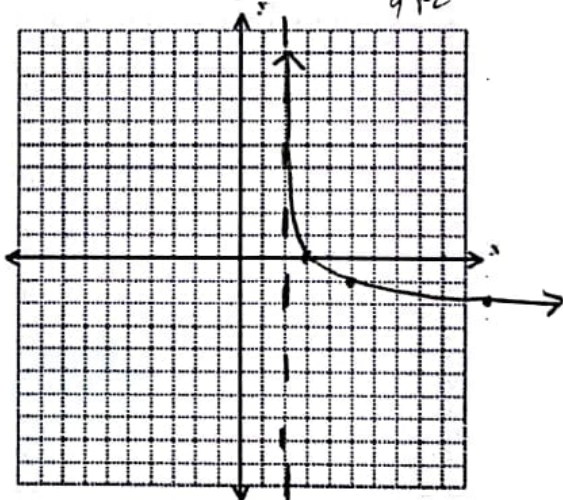


right 2

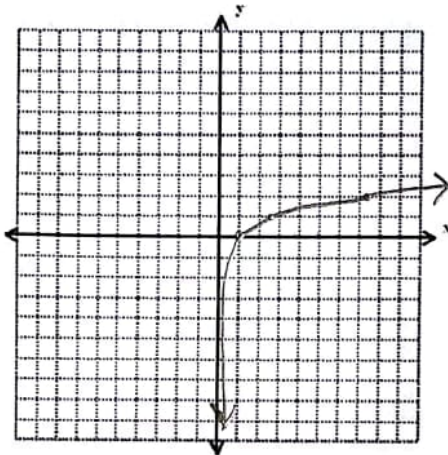
29.  $y = \log_{\frac{1}{3}}(x-2)$

$$y = \log_{\frac{1}{3}} x$$

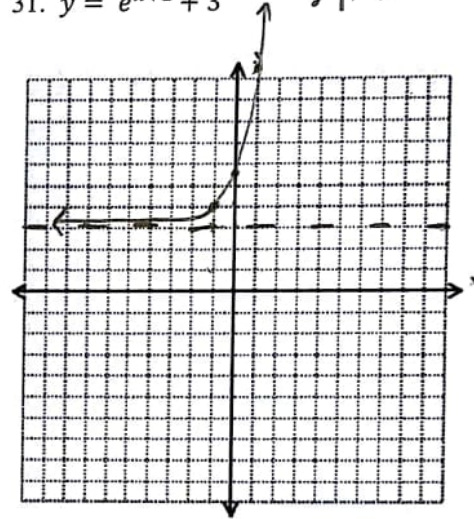
x	y
1	0
3	-1
9	-2



30.  $y = \ln x$



31.  $y = e^{x+1} + 3$



$$(32) 8^x = \frac{1}{64}$$

$$8^x = \frac{1}{8^2}$$

$$8^x = 8^{-2}$$

$$x = -2$$

$$(33) \frac{1}{25}^{x-4} = 5^{7x-3}$$

$$(5^{-2})^{x-4} = 5^{7x-3}$$

$$5^{-2x+8} = 5^{7x-3}$$

$$-2x+8 = 7x-3$$

$$11 = 9x$$

$$\frac{11}{9} = x$$

$$(34) e^{x^2} \cdot e^{5x} = \frac{1}{e^6}$$

$$e^{x^2+5x} = e^{-6}$$

$$x^2+5x = -6$$

$$x^2+5x+6=0$$

$$(x+3)(x+2) = 0$$

$$x = -3 \quad x = -2$$

$$(35) 3^{x+2} = 7$$

$$\ln 3^{x+2} = \ln 7$$

$$(x+2)\ln 3 = \ln 7$$

$$x+2 = \frac{\ln 7}{\ln 3}$$

$$x = \frac{\ln 7}{\ln 3} - 2$$

$$(36) 2^{5x-1} = 6^{x+4}$$

$$\ln 2^{5x-1} = \ln 6^{x+4}$$

$$(5x-1)\ln 2 = (x+4)\ln 6$$

$$5x\ln 2 - \ln 2 = x\ln 6 + 4\ln 6$$

$$5x\ln 2 - x\ln 6 = 4\ln 6 + \ln 2$$

$$x(5\ln 2 - \ln 6) = 4\ln 6 + \ln 2$$

$$x = \frac{4\ln 6 + \ln 2}{5\ln 2 - \ln 6}$$

$$(37) \log_x \frac{1}{64} = 2$$

$$x^2 = \frac{1}{64}$$

$$x = \pm \sqrt{\frac{1}{64}}$$

$$x = \pm \frac{1}{8}$$

$$x = \frac{1}{8}$$

$$(38) \log_3(7x+3) = \log_3(5x+9)$$

$$7x+3 = 5x+9$$

$$2x = 6$$

$$x = 3$$

$$(39) \log_7(x-2) + \log_7(x+3) = \log_7 14$$

$$\log_7(x^2+x-6) = \log_7 14$$

$$x^2+x-6 = 14$$

$$x^2+x-20=0$$

$$(x+5)(x-4)=0$$

$$x = -5 \quad x = 4$$

↳ Extraneous

$$(40) \log_2(5x+7) = 5$$

$$2^5 = 5x+7$$

$$32 = 5x+7$$

$$25 = 5x$$

$$x = 5$$

$$(41) \ln(4x-1) = 3$$

$$e^3 = 4x-1$$

$$e^3+1 = 4x$$

$$\frac{e^3+1}{4} = x$$

$$(42) \log_4 x + \log_4(x-12) = 3$$

$$\log_4(x^2-12x) = 3 \quad | \quad 0 = (x-16)(x+4)$$

$$4^3 = x^2-12x$$

$$0 = x^2-12x-64$$

$$x = 16 \quad x = -4$$

↳ Extraneous

$$(43) \log_4(2x+1) = \log_4(x+2) - \log_4 3$$

$$\log_4(2x+1) = \log_4\left(\frac{x+2}{3}\right)$$

$$2x+1 = \frac{x+2}{3}$$

$$6x+3 = x+2$$

$$5x = -1$$

$$x = -\frac{1}{5}$$

$$(44) \log_2(x+1) - \log_2(x-4) = 3$$

$$\log_2\left(\frac{x+1}{x-4}\right) = 3$$

$$2^3 = \frac{x+1}{x-4}$$

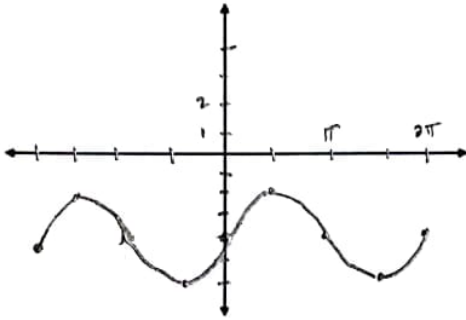
$$8x-32 = x+1$$

$$7x = 33$$

$$x = \frac{33}{7}$$

Graph each of the following. Remember to label your x and y axis correctly.

45.  $y = 2 \sin x - 4$  V.S. by 2  
down 4



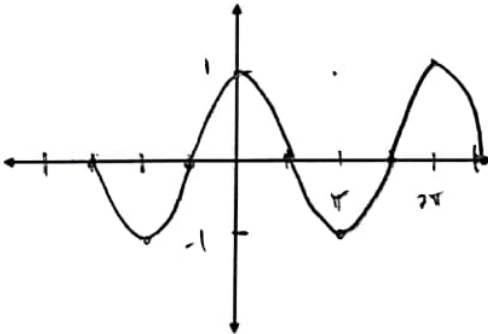
$y = \sin x$

x	y
0	0
$\frac{\pi}{2}$	1
$\pi$	0
$\frac{3\pi}{2}$	-1
$2\pi$	0

$y = 2 \sin x$

x	y
0	0
$\frac{\pi}{2}$	2
$\pi$	0
$\frac{3\pi}{2}$	-2
$2\pi$	0

47.  $y = -\sin(x - \frac{\pi}{2})$  X-axis flip  
right  $\frac{\pi}{2}$



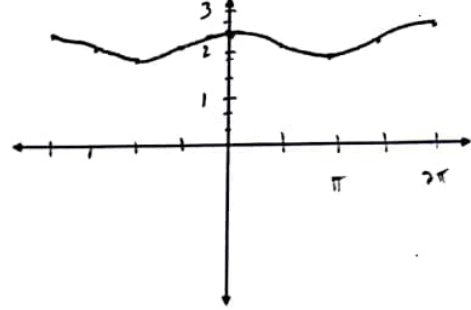
$y = \sin x$

x	y
0	0
$\frac{\pi}{2}$	1
$\pi$	0
$\frac{3\pi}{2}$	-1
$2\pi$	0

$y = -\sin x$

x	y
0	0
$\frac{\pi}{2}$	-1
$\pi$	0
$\frac{3\pi}{2}$	1
$2\pi$	0

46.  $y = \frac{1}{3} \cos x + 2$  V.C. by 3  
up 2



$y = \cos x$

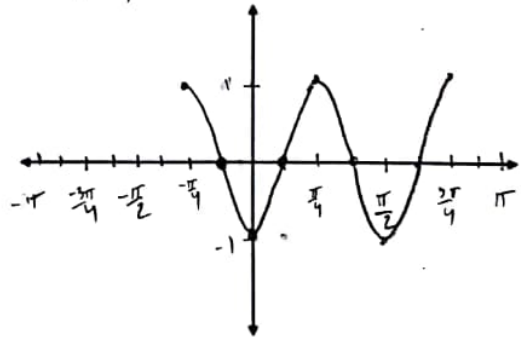
x	y
0	1
$\frac{\pi}{2}$	0
$\pi$	-1
$\frac{3\pi}{2}$	0
$2\pi$	1

$y = \frac{1}{3} \cos x$

x	y
0	$\frac{1}{3}$
$\frac{\pi}{2}$	0
$\pi$	$-\frac{1}{3}$
$\frac{3\pi}{2}$	0
$2\pi$	$\frac{1}{3}$

48.  $y = \cos(4x + \pi) = \cos[4(x + \frac{\pi}{4})]$

H.C. by 4  
left  $\frac{\pi}{4}$



$y = \cos x$

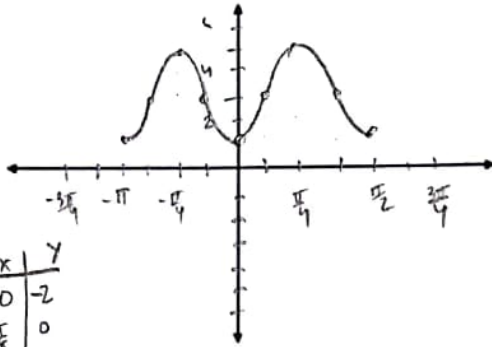
x	y
0	1
$\frac{\pi}{2}$	0
$\pi$	-1
$\frac{3\pi}{2}$	0
$2\pi$	1

x	y
0	1
$\frac{\pi}{4}$	0
$\frac{\pi}{2}$	-1
$\frac{3\pi}{4}$	0
$\pi$	1



x-axis Flip H.C. by 4  
 v.s by 2 up 3

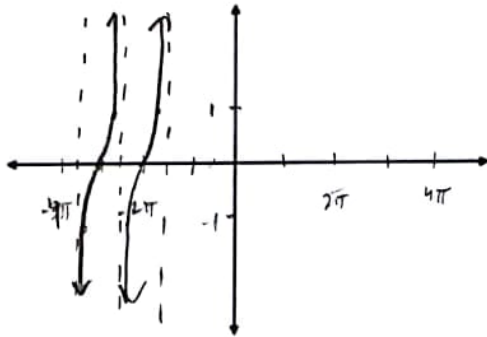
49.  $y = -2 \cos(4x) + 3$



x	y
0	1
$\frac{\pi}{2}$	0
$\pi$	-1
$\frac{3\pi}{2}$	0
$2\pi$	1

x	y
0	-2
$\frac{\pi}{2}$	0
$\pi$	-2
$\frac{3\pi}{2}$	0
$2\pi$	-2

51.  $y = \frac{1}{2} \tan(x + 3\pi) - 1$

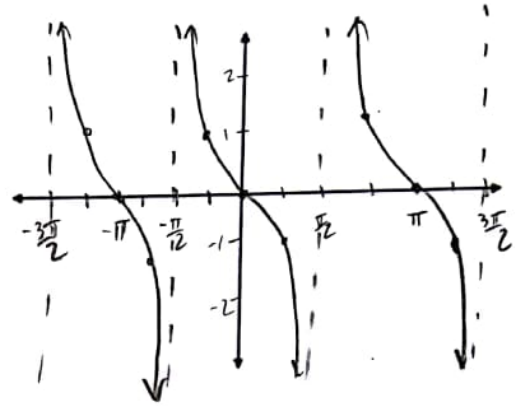


$y = \tan x$

$x = -\frac{\pi}{2}$	$x = \frac{\pi}{2}$
$(-\frac{\pi}{4}, -1)$	$(-\frac{\pi}{4}, \frac{1}{2})$
$(0, 0)$	$(0, 0)$
$(\frac{\pi}{4}, 1)$	$(\frac{\pi}{4}, \frac{1}{2})$
$x = \frac{\pi}{2}$	$x = \frac{\pi}{2}$

x-axis  
 flip

50.  $y = -\tan x$



v.l. by 2

down 1  
 left + 3pi

(52) -/+

(53) (0, -1)

(54) 3

(55) 3

(56) 2

(57) 0

(58) 1

(59) (0, -512)

(60) -/-

(61) 10

(62)  $x = 3, -2, 1$

(63)  $x = 3$  bounce

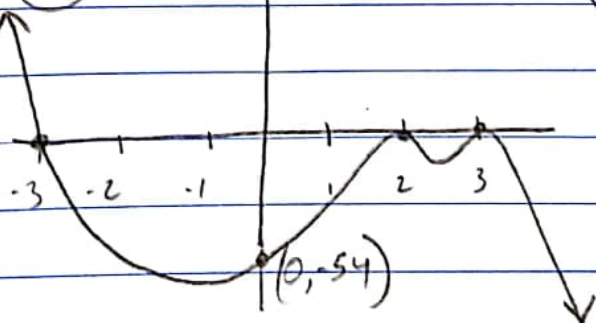
$x = -2$  cross

$x = 1$  cross

(64) 9

(65) 1

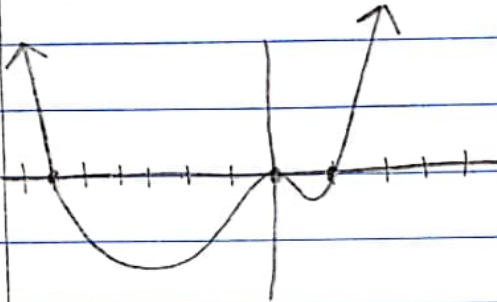
(66)



(67)  $y = x^4 + 5x^3 - 6x^2$

$y = x^2(x^2 + 5x - 6)$

$y = x^2(x+6)(x-1)$



(68)  $y = 2(x-3)^2 + 1$

$y = 2(x^2 - 6x + 9) + 1$

$y = 2x^2 - 12x + 18 + 1$

$y = 2x^2 - 12x + 19$

(69)  $y = -\frac{1}{2}(4x-10)^2 - 14$

$y = -\frac{1}{2}(16x^2 - 80x + 100) - 14$

$y = -8x^2 + 40x - 50 - 14$

$y = -8x^2 + 40x - 64$

(70)  $y = x^2 + 18x + 4$

$y = (x^2 + 18x + 81) + 4 - 81$

$y = (x+9)^2 - 77$

$$(71) y = -2x^2 - 12x - 11$$

$$y = -2(x^2 + 6x + 9) - 11 - (-18)$$

$$y = -2(x+3)^2 + 7$$

$$(72) y = 5x^2 + 60x - 1$$

$$y = 5(x^2 + 12x + 36) - 1 - 180$$

$$y = 5(x+6)^2 - 181$$

$$(73) y = x^2 + 7x$$

$$y = (x^2 + 7x + \frac{49}{4}) - \frac{49}{4}$$

$$y = (x + \frac{7}{2})^2 - \frac{49}{4}$$

$$(74) y = 3x^2 + 9x - 12$$

$$y = 3(x^2 + 3x + \frac{9}{4}) - 12 - \frac{27}{4}$$

$$y = 3(x + \frac{3}{2})^2 - \frac{75}{4}$$

$$(75) C = (0, 0) \quad r = 5$$

$$(76) C = (2, -5) \quad r = 4$$

$$(77) C = (-4, 0) \quad r = \sqrt{11}$$

$$(78) (x-5)^2 + (y+3)^2 = 16$$

$$(79) x^2 + (y-2)^2 = 5$$

$$(80) C = (4, 11) \quad r^2 = (5+4)^2 + (-1-11)^2$$

$$r^2 = 81 + 144$$

$$r^2 = 225$$

$$(x+4)^2 + (y-11)^2 = 225$$

$$(81) (x-3)^2 + y^2 = 5$$

$$x^2 - 6x + 9 + y^2 = 5$$

$$x^2 + y^2 - 6x + 4 = 0$$

$$(82) x^2 + 4x + y^2 - 10y - 11 = 0$$

$$(x^2 + 4x + 4) + (y^2 - 10y + 25) = 11 + 4 + 25$$

$$(x+2)^2 + (y-5)^2 = 40$$

$$(83) x^2 + 24x + y^2 + 10y + 160 = 0$$

$$(x^2 + 24x + 144) + (y^2 + 10y + 25) = -160 + 144 + 25$$

$$(x+12)^2 + (y+5)^2 = 9$$

$$(84) C = \left( \frac{-17+19}{2}, \frac{-9+9}{2} \right)$$

$$C = (-1, 0)$$

$$r^2 = (-14+18)^2 + (-9+9)^2$$

$$r^2 = 16 + 0$$

$$r^2 = 16$$

$$r = 4$$

$$(x+18)^2 + (y+9)^2 = 1$$

$$(85) C = (-10, -3)$$

$$C = 8\pi = 2\pi r$$

$$4 = r$$

$$(x+10)^2 + (y+3)^2 = 16$$

$$(86) C = (3, -4)$$

$$A = 49\pi = \pi r^2$$

$$49 = r^2$$

$$7 = r$$

$$(x-3)^2 + (y+4)^2 = 49$$

$$(87) x^2 + y^2 = 1$$

$$(88) (-6, -3) \cup (1, 5) \cup (9, 10)$$

$$(89) (-9, -6) \cup (-3, 1) \cup (5, 9)$$

$$(90) (-3, 6) \dot{\cup} (5, 8)$$

$$(91) (-6, -3) \dot{\cup} (1, 2) \dot{\cup} (9, 5)$$

$$(92) [-9, 10]$$

$$(93) [-5, 8]$$

$$(95) (-8, -5) \cup (-2, \infty)$$

$$(96) (-5, -2)$$

$$(97) (-5, 2)$$

$$(98) (-2, -1)$$

$$(99) (-8, \infty)$$

$$(100) [-1, \infty)$$

$$(101) y = \frac{1}{30}(x+3)(x-2)^2(x-5)$$

$$(102) y = -\frac{1}{2}(x+1)^2(x-2)(x-4)$$

$$(103) y = -\frac{1}{50}(x+5)(x+2)(x-2)(x-5)$$

$$(104) y = -\frac{1}{9}(x+4)(x-3)^3$$