2018 Midterm Review

(1) $\left(\frac{f(x+h)-f(x)}{h}\right)$ slope of a secant line
or Average Rate of change

(2) Limit

(3) (a) f(x) is discontinuous at x=a

(b) f(x) has a sharp point at x=a

(c) If x=a is the endpoint of f(x)

Don't really need to know this one: (d) If f(x) oscillates between 2 fixed values as x approaches a

(e) f(x) has a vertical tangent at x=a

(4) Derivative

(5) hole

(6) Horizontal asymptotes
&
End behavior

(7) Slope of a secant line between the points (x, f(x)) & (x+h, f(x+h))
or Average Rate of change
(Same as #1)

(8) slope of a tangent line
or slope of the graph

(9) Indeterminate

(10) at x=1 (sharp point)

(11) vertical asymptote

(12) slope of a secant line

(13) at x=3 (vertical tangent line)

(14) $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

The slope of a secant line where the 2 points are becoming infinitely close together.

(15) There is a vertical asymptote at the x-value that was plugged in

(16) a point on the graph of a function where there is a hole, break or vertical asymptote

(17) $\frac{f(3)-f(-1)}{3-(-1)} = \frac{(6-27)-(-2-3)}{3+1} = \frac{-21+5}{4}$

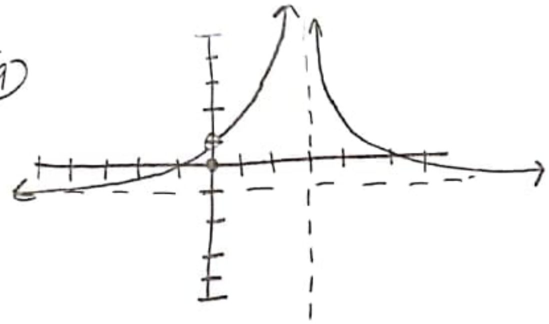
$\frac{-16}{4} = -4\ m_{secant}$

(18) $m_{tan} = f'(-1)$

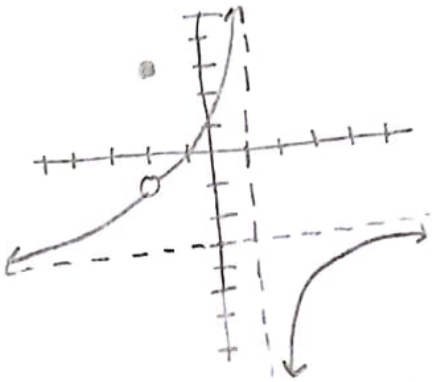
$f'(x) = 2-6x$

$f'(-1) = 2-6(-1) = 2+6 = 8 = m_{tan}$

(19)



(20)



(21) $\lim_{x\to -2^-} \left(\frac{1}{x+2}\right) = \frac{1}{-2+2} = \frac{1}{0}$

$\frac{+}{-1.9+2} = \frac{+}{-} = -\infty$

(22) $\lim_{x\to\infty} \left(\frac{3x^2+2x+1}{2-x^2}\right) = \frac{3x^2}{-x^2} = -3$

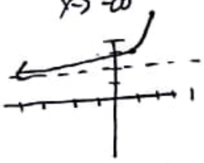
(23) $\lim_{x\to 3} \left(\frac{4x^2-21x+27}{9+x^2}\right) = \frac{36-63-27}{9+9}$

$\frac{0}{18} = 0$

(24) $\lim_{x\to 2} \left(\frac{x-2}{x^3-8}\right) = \frac{\cancel{x-2}}{\cancel{(x-2)}(x^2+2x+4)}$

$= \frac{1}{x^2+2x+4} = \frac{1}{4+4+4} = \frac{1}{12}$

(25) $\lim_{x\to -\infty} (e^{x-1}+2) = 2$



(26) $\lim_{x\to\infty} \frac{2x+1}{3-5x+x^2} = \frac{2x}{x^2} = \frac{2}{x} = 0$

(27) $\lim_{x\to 2} (4x^2-3x+1) = 16-6+1 = 11$

(28) $\lim_{x\to\infty} \left(\frac{3x^2+2x+1}{2-x^2}\right) = \frac{3x^2}{-x^2}$ -3

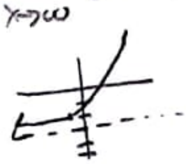
(29) $\lim_{x\to 1} \frac{3x^2-2x-1}{1-x^4} = \frac{3-2-1}{1-1} = \frac{0}{0}$

$\frac{(3x+1)\cancel{(x-1)}}{-1\cancel{(x-1)}(x+1)(x^2+1)} = \frac{3+1}{(-1)(2)(2)} = \frac{-4}{4} = -1$

(30) $\lim_{x\to -\infty} \frac{5x^3x}{7-2x} = \frac{5x^3}{-2x} = \frac{5x^2}{-2}$

$-\infty$

(31) $\lim_{x\to\infty} (2^{x+1}-3) = \infty$



(32) $\lim_{x\to 13} \frac{\sqrt{x-4}-3}{x-13} \xrightarrow{} \frac{0}{0}$ $\frac{\sqrt{x-4}+3}{\sqrt{x-4}+3} = \frac{x-4-9}{(x-13)(\sqrt{x-4}+3)}$

$\lim_{x\to 13} \frac{\cancel{(x-13)}}{\cancel{(x-13)}(\sqrt{x-4}+3)} = \lim_{x\to 13} \frac{1}{\sqrt{x-4}+3} = \frac{1}{6}$

(33) $\lim_{x\to 2} \frac{x+1}{(x-2)^2} = \frac{3}{0} = \frac{+}{+} = +\infty$

(34) $\lim_{x\to 2} \frac{\frac{x-2}{x+3}}{x^3-2x^2+4x-8} \xrightarrow{} \frac{0}{0}$ $= \left(\frac{x-2}{x+3}\right)\left(\frac{1}{x^2(x-2)+4(x-2)}\right)$

$\lim_{x\to 2} \left(\frac{\cancel{x-2}}{x+3}\right)\left(\frac{1}{(x^2+4)\cancel{(x-2)}}\right) = \frac{1}{(5)(8)} = \frac{1}{40}$

(35) $\lim_{h\to 0} \frac{\frac{1}{2+h}-\frac{1}{2}}{h} \xrightarrow{} \frac{0}{0}$ $= \frac{2-(2+h)}{4+2h} \cdot \frac{1}{h} = \frac{\cancel{2}-\cancel{2}-h}{h(4+2h)}$

$\lim_{h\to 0} \frac{-1}{4+2h} = -\frac{1}{4}$

36) $\lim_{x\to -\infty} (3x^2 - 3x^5) = \infty$

37) $\lim_{x\to \infty} (3x^2 - 3x^5) = -\infty$

38) $\lim_{x\to \infty} \frac{x^2+4x+4}{x^2-4} = \frac{x^2}{x^2} = 1$

39) $\lim_{x\to 2} \frac{x^2-4x+4}{x^2-4} = \frac{4-8+4}{4-4} = \frac{0}{0}$

$\frac{(x-2)(x-2)}{\cancel{(x-2)}(x+2)} = \frac{0}{4} = 0$

40) $\lim_{x\to -2} \frac{x^2-4x+4}{x^2-4} = \frac{4+8+4}{4-4} = \frac{16}{0}$

$\lim_{x\to -2^-} \frac{+}{(-)(-)} = +\infty$

$\lim_{x\to -2^+} \frac{+}{(-)(+)} = \frac{+}{-} = -\infty$

$\lim_{x\to -2} f(x) = \text{DNE}$

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41) The IROC or slope of the tangent line at $x=2$ for $f(x)=\frac{1}{x}$

or $f'(2)$ for $f(x)=\frac{1}{x}$

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42) $\frac{d}{dx} k = 0$

43) $\frac{d}{dx} x^n = nx^{n-1}$

44) $\frac{d}{dx} k f(x) = k f'(x)$

45) $\frac{d}{dx} f(x) \pm g(x) = f'(x) \pm g'(x)$

46) $\frac{d}{dx} f(x)g(x) = f(x)g'(x) + f'(x)g(x)$

47) $\frac{d}{dx} \frac{T(x)}{B(x)} = \frac{B(x)T'(x) - B'(x)T(x)}{(B(x))^2}$

48) $\frac{d}{dx} [f(x)]^n = n[f(x)]^{n-1} f'(x)$

49) $\frac{d}{dx} \ln x = \frac{1}{x}$ or x^{-1}

50) $\frac{d}{dx} e^x = e^x$

51) $\frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}$

52) $\frac{d}{dx} e^{f(x)} = (e^{f(x)})(f'(x))$

53) $\frac{d}{dx} \log_b f(x) = \left(\frac{1}{\ln b}\right)\left(\frac{f'(x)}{f(x)}\right)$

54) $\frac{d}{dx} b^{f(x)} = (\ln b)(b^{f(x)})(f'(x))$

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55) $g(x) = 7e^2 + \pi - 3 \Rightarrow \underline{g'(x) = 0} \Rightarrow g'(1) = 0$

56) $h(x) = (3x-5)^2 = 9x^2 - 30x - 25 \Rightarrow \underline{h'(x) = 18x - 30} \Rightarrow h'(-2) = -36 - 30 = -66$

(57) $h(x) = (11x)^{1/2} \Rightarrow h'(x) = \frac{1}{2}(11x)^{-1/2}(11) \Rightarrow h'(11) = \frac{11}{2\sqrt{121}} = \frac{11}{2 \cdot 11} = \frac{1}{2}$

$h'(x) = \frac{11}{2\sqrt{11x}}$

or

$h(x) = \sqrt{11}\, x^{1/2} \Rightarrow h'(x) = \frac{\sqrt{11}}{2} x^{-1/2} \Rightarrow h'(11) = \frac{\sqrt{11}}{2\sqrt{11}} = \frac{1}{2}$

$h'(x) = \frac{\sqrt{11}}{2\sqrt{x}}$

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(58) $d(x) = \frac{3x^3 - 2x + 7}{x^2} = d(x) = 3x - 2x^{-1} + 7x^{-2} \Rightarrow d'(x) = 3 + 2x^{-2} - 14x^{-3}$

$d'(x) = 3 + \frac{2}{x^2} - \frac{14}{x^3}$

$d'(-1) = 3 + \frac{2}{1} - \frac{14}{-1} = 3 + 2 + 14 = 19$

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(59) $r(x) = \left(2x^{-1} - \frac{1}{2}x^{\frac{1}{4}}\right)^{1/3} \Rightarrow r'(x) = \frac{1}{3}\left(2x^{-1} - \frac{1}{2}x^{\frac{1}{4}}\right)^{-2/3}\left(-2x^{-2} - \frac{1}{8}x^{-3/4}\right)$

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(60) $f(x) = 3(6x^7 + 5x^3 + 7x)^{-4} \Rightarrow f'(x) = -12(6x^7 + 5x^3 + 7x)^{-5}(42x^6 + 15x^2 + 7)$

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(61) $f(x) = \left(\frac{3}{2}x^{-1} + \frac{1}{3}x\right)(x^3 + 5x)^{1/2}$

$f'(x) = \left(\frac{3}{2}x^{-1} + \frac{1}{3}x\right)\left[\frac{1}{2}(x^3+5x)^{-1/2}(3x^2+5)\right] + \left(-\frac{3}{2}x^{-2} + \frac{1}{3}\right)(x^3+5x)^{1/2}$

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(62) $f(x) = \frac{3}{5}x^{-1} + \frac{2}{7}x^2 + e^3 + x^e \Rightarrow f'(x) = -\frac{3}{5}x^{-2} + \frac{4}{7}x + 0 + ex^{e-1}$

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(63) $f(x) = \left(\frac{1-3x}{2x^2+7x}\right)\left(\frac{x+5}{2x}\right) = \frac{x + 5 - 3x^2 - 15x}{4x^3 + 14x} = \frac{-3x^2 - 14x + 5}{4x^3 + 14x}$

$f'(x) = \frac{(4x^3+14x)(-6x-14) - (12x^2+14)(-3x^2-14x+5)}{(4x^3+14x)^2}$

or $f'(x) = \left(\frac{1-3x}{2x^2+7x}\right)\left[\frac{(2x)(1) - (2)(x+5)}{(2x)^2}\right] + \left[\frac{(2x^2+7x)(-3) - (4x+7)(1-3x)}{(2x^2+7x)^2}\right]\left(\frac{x+5}{2x}\right)$

(64) $p(x) = (5-5x^2)^{1/2}\left(\frac{6+x^2}{2x-6x^3}\right)$

$$p'(x) = (5-5x^2)^{1/2}\left[\frac{(2x-6x^3)(2x)-(2-18x^2)(6+x^2)}{(2x-6x^3)^2}\right] + \left[\frac{1}{2}(5-5x^2)^{-1/2}(-10x)\right]\left(\frac{6+x^2}{2x+6x^3}\right)$$

(65) $h(x) = \dfrac{\frac{1}{2}x^{-1}+5x^{1/3}}{8x-7} \Rightarrow h'(x) = \dfrac{(8x-7)\left(-\frac{1}{2}x^{-2}+\frac{5}{3}x^{-2/3}\right)-(8)\left(\frac{1}{2}x^{-1}+5x^{1/3}\right)}{(8x-7)^2}$

(66) $r(x) = \left(\dfrac{1-x^{1/2}}{\left(\frac{4x^2}{3}\right)}\right)\left(\dfrac{2e+3}{\pi x}\right) = \dfrac{(2e+3)(1-x^{1/2})}{\left(\frac{4\pi}{3}\right)x^3} = \left(\frac{3}{4\pi}\right)(2e+3)(1-x^{1/2})(x^{-3})$

$r(x) = \left(\dfrac{6e+9}{4\pi}\right)\left(x^{-3}-x^{-5/2}\right) \Rightarrow r'(x) = \left(\dfrac{6e+9}{4\pi}\right)\left(-3x^{-4}+\frac{5}{2}x^{-7/2}\right)$

or

$$r'(x) = \left(\frac{1-x^{1/2}}{x^2-\frac{x^2}{3}}\right)\left[\frac{(\pi x)(0)-(\pi)(2e+3)}{(\pi x)^2}\right] + \left[\frac{\left(x^2-\frac{x^2}{3}\right)\left(-\frac{1}{2}x^{-1/2}\right)-\left(2x-\frac{2x}{3}\right)(1-x^{1/2})}{\left(x^2-\frac{x^2}{3}\right)^2}\right]$$

(67) $h(x) = \left(\dfrac{6ex+x^2-x}{x^{2/3}-3x^3}\right)^4 = (6ex+x^2-x)^4(x^{2/3}-3x^3)^{-4}$

$$h'(x) = 4\left(\frac{6ex+x^2-x}{x^{2/3}-3x^3}\right)^3\left[\frac{(x^{2/3}-3x^3)(6e+2x-1)-\left(\frac{2}{3}x^{-1/3}-9x^2\right)(6ex+x^2-x)}{(x^{2/3}-3x^3)^2}\right]$$

or

$$h'(x) = (6ex+x^2-x)^4\left[-4(x^{2/3}-3x^3)^{-5}\left(\tfrac{2}{3}x^{-1/3}-9x^2\right)\right] + \left[4(6ex+x^2-x)^3(6e+2x-1)\right](x^{2/3}-3x^3)^{-4}$$

(68) $f(x) = \left[\left(\frac{1}{2}x^{-1}+3x^4\right)\left(x^{3/5}+x^5-x^{-4}\right)\right]^5 = \left(\frac{1}{2}x^{-1}+3x^4\right)^5\left(x^{3/5}+x^5-x^{-4}\right)^5$

$$f'(x) = 5\left[\left(\tfrac{1}{2}x^{-1}+3x^4\right)\left(x^{3/5}+x^5-x^{-4}\right)\right]^4\left[\left(\tfrac{1}{2}x^{-1}+3x^4\right)\left(\tfrac{3}{5}x^{-2/5}+5x^4+4x^{-5}\right)+\left(-\tfrac{1}{2}x^{-2}+12x^3\right)\left(x^{3/5}+x^5-x^{-4}\right)\right]$$

or

$$f'(x) = \left(\tfrac{1}{2}x^{-1}+3x^4\right)^5\left[5\left(x^{3/5}+x^5-x^{-4}\right)^4\left(\tfrac{3}{5}x^{-2/5}+5x^4+4x^{-5}\right)\right] + \left[5\left(\tfrac{1}{2}x^{-1}+3x^4\right)^4\left(-\tfrac{1}{2}x^{-2}+12x^3\right)\right]\left(x^{3/5}+x^5-x^{-4}\right)^5$$

(69) $g(x) = \left[\ln(2x^2+3)^5\right]^{1/3} = \left[5\ln(2x^2+3)\right]^{1/3}$

$g'(x) = \frac{1}{3}\left[5\ln(2x^2+3)\right]^{-2/3}\left[\frac{5(4x)}{2x^2+3}\right]$

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(70) $r(x) = \log\left(\frac{x^3-7}{10^{2x}}\right) = \log(x^3-7) - \log 10^{2x} = \log(x^3-7) - 2x$

$r'(x) = \left(\frac{1}{\ln 10}\right)\left(\frac{3x^2}{x^3-7}\right) - 2$

or

$r'(x) = \dfrac{\dfrac{(10^{2x})(3x^2) - [(\ln 10)(10^{2x})(2)](x^3-7)}{(10^{2x})^2}}{\dfrac{x^3-7}{10^{2x}}}$

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(71) $v(x) = \left[\ln\left(\frac{x}{5}+\frac{2}{x}\right) - \ln(3x\,e^x)\right]^{1/2} = \left[\ln\left(\frac{1}{5}x+2x^{-1}\right) - \ln 3 - \ln x - x\right]^{1/2}$

$v'(x) = \frac{1}{2}\left[\ln\left(\frac{\frac{x}{5}+\frac{2}{x}}{3x\,e^x}\right)\right]^{-1/2}\left[\left(\frac{\frac{1}{5}-2x^{-2}}{\frac{1}{5}x+2x^{-1}}\right) - 0 - \frac{1}{x} - 1\right]$

or

$v'(x) = \frac{1}{2}\left[\ln\left(\frac{\frac{x}{5}+\frac{2}{x}}{3x\,e^x}\right)\right]^{-1/2}\left[\dfrac{\dfrac{(3x\,e^x)\left(\frac{1}{5}-2x^{-2}\right) - [3x\,e^x + 3e^x]\left(\frac{x}{5}+\frac{2}{x}\right)}{(3x\,e^x)^2}}{\dfrac{\frac{x}{5}+\frac{2}{x}}{3x\,e^x}}\right]$

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(72) $d(x) = (x^{1/3} - x\ln x)(\log_2 7)$

$d'(x) = (\log_2 7)\left[\frac{1}{3}x^{-2/3} - \left((x)\left(\frac{1}{x}\right) + (1)(\ln x)\right)\right]$

or

$d'(x) = \left(\frac{1}{\ln 2}\right)\dfrac{(\ln 7)\left(7^{x^{1/3} - x\ln x}\right)\left[\frac{1}{3}x^{-2/3} - \left((x)\left(\frac{1}{x}\right) + (1)(\ln x)\right)\right]}{7^{x^{1/3}} - x\ln x}$

(73) $$f'(x)=\frac{\left[\log_4\left(e^{3x}+e^{1/2}\right)\right]\left[\left(\frac{1}{\ln 4}\right)\left(\frac{2x-7}{x^2-7x}\right)\right]-\left[\left(\frac{1}{\ln 4}\right)\left(\frac{3e^{3x}}{e^{3x}+e^{1/2}}\right)\right]\left[\log_4\left(x^2-7x\right)\right]}{\left[\log_4\left(e^{3x}+e^{1/2}\right)\right]^2}$$

(74) $$g(x)=\frac{\cancel{3}}{\cancel{3}\ln\left(\frac{3}{2}x^{-1}-3^x\right)}=\left[\ln\left(\frac{3}{2}x^{-1}-3^x\right)\right]^{-1}$$

$$g'(x)=-1\left[\ln\left(\frac{3}{2}x^{-1}-3^x\right)\right]^{-2}\left[\frac{-\frac{3}{2}x^{-2}-(\ln 3)(3^x)}{\frac{3}{2}x^{-1}-3^x}\right]$$

or $$g'(x)=\frac{0-\left(\frac{-\frac{3}{2}x^{-2}-(\ln 3)(3^x)}{\frac{3}{2}x^{-1}-3^x}\right)(1)}{\left[\ln\left(\frac{3}{2}x^{-1}-3^x\right)\right]^2}$$

(75) $$k(x)=\log 3-2\ln 3x^{-1/2}+(\ln x)^5=\log 3-2\left(\ln 3+\ln x^{-1/2}\right)+(\ln x)^5$$

$$k(x)=\log 3-2\ln 3+\ln x+(\ln x)^5$$

$$k'(x)=0-0+\frac{1}{x}+5(\ln x)^4\left(\frac{1}{x}\right)$$

(76) $$h(x)=\log\left[\ln\left(4x^{-5}+5^{2x}\right)^3\right]^{1/2}=\frac{1}{2}\log\left[3\ln\left(4x^{-5}+5^{2x}\right)\right]$$

$$h'(x)=\left(\frac{1}{2}\right)\left(\frac{1}{\ln 10}\right)\left[\frac{(3)\left(\frac{-20x^{-6}+(\ln 5)(5^{2x})(2)}{4x^{-5}+5^{2x}}\right)}{3\ln\left(4x^{-5}+5^{2x}\right)}\right]$$

(77) $$f(x)=\left[\frac{x}{(5x-3)^2}\right]^{1/3}=\frac{x^{1/3}}{(5x-3)^{2/3}}=\left(x^{1/3}\right)\left(5x-3\right)^{-2/3}$$

$$f'(x)=\frac{1}{3}\left[\frac{x}{(5x-3)^2}\right]^{-2/3}\left[\frac{(5x-3)^2(1)-\left(2(5x-3)(5)\right)(x)}{(5x-3)^4}\right]$$

or $$f'(x)=\frac{(5x-3)^{2/3}\left(\frac{1}{3}x^{-2/3}\right)-\left[\frac{2}{3}(5x-3)^{-1/3}(5)\right]\left(x^{1/3}\right)}{\left[(5x-3)^{2/3}\right]^2}$$

or $$f'(x)=\left(x^{1/3}\right)\left[-\frac{2}{3}(5x-3)^{-5/3}(5)\right]+\left(\frac{1}{3}x^{-2/3}\right)\left(5x-3\right)^{-2/3}$$

(78) $g(x) = \dfrac{(x)(2^{x^2-4x})}{\log 3 + \log x + \log e^x} = \dfrac{(x)(2^{x^2-4x})}{\log 3 + \log x + x\log e}$

$$g'(x) = \frac{\left[\log(3xe^x)\right]\left[(x)(2^{x^2-4x})(\ln 2)(2x-4) + (1)(2^{x^2-4x})\right] - \left[\frac{1}{x} + \log e\right](x)(2^{x^2-4x})}{\left(\log 3xe^x\right)^2}$$

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(79) $f(x) = (2x)(10-3x)^4\left(\ln 3 + 2\ln x - e^{2-x}\right)$

$$f'(x) = (2x)(10-3x)^4\left[\frac{2}{x} - (e^{2-x})(-1)\right] + (2x)\left[4(10-3x)^3(-3)\right]\left(\ln 3x^2 - e^{2-x}\right) + (2)(10-3x)^4\left(\ln 3x^2 - e^{2-x}\right)$$

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(80) $f'(0) = 0$

(81) $f'(-4) =$ D.N.E

(82) $f(4) =$ D.N.E

(83) $f'(3) =$ D.N.E

(84) $f'(4) =$ D.N.E

(85) 1

(86) 1

(87) ∞

(88) $-\infty$

(89) 3

(90) -1

(91) 2

(92) $x = -2$, $x = 3$, $x = 4$

(93) $x = 4$, $x = -2$, $x = 3$, $x = 4$

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(94) $f(x) = \sqrt{x} - 1$

$f(x+h) = \sqrt{x+h} - 1$

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{(\sqrt{x+h} - 1) - (\sqrt{x} - 1)}{h}$$

$$\frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$\frac{(\cancel{x}+\cancel{h}) - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$\lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$\frac{1}{2\sqrt{x}} = f'(x)$$

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(95) $f(x) = 2x - x^2$

$f(x+h) = 2(x+h) - (x+h)^2$

$= 2x + 2h - x^2 - 2xh - h^2$

$$\frac{(\cancel{2x} + 2h - \cancel{x^2} - 2xh - h^2) - (\cancel{2x} - \cancel{x^2})}{h}$$

$$\frac{2h - 2xh - h^2}{h}$$

$2 - 2x - h$

$$\lim_{h \to 0}(2 - 2x - h)$$

$f'(x) = 2 - 2x$

(96) $f(x) = 3x+5$

$f(x+h) = 3x+3h+5$

$$\frac{(\cancel{3x}+3h+\cancel{5}) - (\cancel{3x}+\cancel{5})}{h}$$

$$\frac{3h}{h} = 3$$

$$\lim_{h\to 0} 3 = 3 = f'(x)$$

(97) $f(x) = 2x^2$

$f(x+h) = 2(x+h)^2$

$= 2(x^2+2xh+h^2)$

$= 2x^2+4xh+2h^2$

$$\frac{(\cancel{2x^2}+4xh+2h^2)-(\cancel{2x^2})}{h}$$

$$\frac{\cancel{h}(4x+2h)}{\cancel{h}}$$

$$\lim_{h\to 0}(4x-2h)$$

$f'(x) = 4x$

(98) $f(x) = 18$

$f(x+h) = 18$

$$\frac{f(x+h)-f(x)}{h}$$

$$\frac{18-18}{h} = \frac{0}{h} = 0$$

$$\lim_{h\to 0} 0 = 0 = f'(x)$$

(99) $f(x) = \frac{2x}{x+5}$

$f(x+h) = \frac{2x+2h}{x+h+5}$

$$\frac{\left(\frac{2x+2h}{x+h+5}\right)-\left(\frac{2x}{x+5}\right)}{h}$$

$$\frac{(2x+2h)(x+5)-(2x)(x+h+5)}{(h)(x+h+5)(x+5)}$$

$$\frac{\cancel{2x^2}+\cancel{2xh}+\cancel{10x}+10h-\cancel{2x^2}-\cancel{2xh}-\cancel{10x}}{\cancel{h}(x+h+5)(x+5)}$$

$$\lim_{h\to 0}\frac{10}{(x+h+5)(x+5)}$$

$$\frac{10}{(x+5)(x+5)} = \frac{10}{(x+5)^2} = f(x)$$

(100) $f(x) = x^3-x^2+5x-8$

$f(x+h) = (x+h)^3-(x+h)^2+5(x+h)-8$

$f(x+h) \cdot x^3+3x^2h+3xh^2+h^3-x^2-2xh-h^2+5x+5h-8$

$$\frac{(\cancel{x^3}+3x^2h+3xh^2+\cancel{h^3}-\cancel{x^2}-2xh-\cancel{h^2}+\cancel{5x}+5h-\cancel{8})-(\cancel{x^3}-\cancel{x^2}+\cancel{5x}-\cancel{8})}{h}$$

$$\frac{3x^2h+3xh^2-2xh+5h}{h} = 3x^2+3xh-2x+5$$

$$\lim_{h\to 0}(3x^2+3xh-2x+5)$$

$3x^2-2x+5 = f'(x)$

(101) $\lim_{x\to -\frac{3\pi}{4}} \left[(\csc x)(\tan x)\right] = \csc\left(\frac{5\pi}{4}\right)\tan\left(\frac{3\pi}{4}\right) = \left(\csc\frac{5\pi}{4}\right)\left(\tan\frac{5\pi}{4}\right) = (-\sqrt{2})(1) = -\sqrt{2}$

(102) $\lim_{x\to 0} \frac{\tan^2 x}{x} \rightarrow \frac{0}{0} = \lim_{x\to 0}\left(\frac{\sin x}{x}\right)\left(\frac{\sin x}{\cos^2 x}\right) = (1)\left(\frac{0}{1}\right) = 0$

(103) $\lim_{x\to 0} \frac{x^2}{\sin^2 x} \rightarrow \frac{0}{0} = \lim_{x\to 0}\left(\frac{x}{\sin x}\right)\left(\frac{x}{\sin x}\right) = (1)(1) = 1$

(104) $\lim_{x\to \frac{\pi}{4}} \frac{\sec^2 x - \csc^2 x}{\tan x - 1} \rightarrow \frac{0}{0} = \frac{\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}}{\frac{\sin x}{\cos x} - 1} = \frac{\sin^2 x - \cos^2 x}{(\cos^2 x)(\sin^2 x)} \cdot \frac{\cancel{\cos x}}{\sin x - \cos x}$

$= \frac{\cancel{(\sin x - \cos x)}(\sin x + \cos x)}{(\cos x)(\sin^2 x)\cancel{(\sin x - \cos x)}} = \frac{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}}{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)} = \left(\frac{\sqrt{2}}{1}\right)\left(\frac{4}{\sqrt{2}}\right) = 4$

(105) $\lim_{x\to \frac{\pi}{4}^+} \frac{\sin^2 x}{\cos(2x)} = \frac{\left(\frac{\sqrt{2}}{2}\right)^2}{0} = \frac{+}{-} = -\infty$

(106) $\lim_{x\to \frac{\pi}{2}} \frac{\cos(2x) + \sin^2 x}{\csc^2 x - 1} = \frac{-1+1}{1-1} = \frac{0}{0}$

$\lim_{x\to \frac{\pi}{2}} \frac{\cos^2 x - \cancel{\sin^2 x} + \cancel{\sin^2 x}}{\cot^2 x} = \frac{\cos^2 x}{\frac{\cos^2 x}{\sin^2 x}} = \frac{\cancel{\cos^2 x}}{1} \cdot \frac{\sin^2 x}{\cancel{\cos^2 x}} = \sin^2 x = (1)^2 = 1$

(107) $\lim_{x\to 2} \frac{\sin(x-2)}{x^2+2x-8} \rightarrow \frac{0}{0} = \lim_{x\to 2} \frac{\sin(x-2)}{(x-2)} \cdot \frac{1}{(x+4)} = (1)\left(\frac{1}{6}\right) = \frac{1}{6}$

(108) $\lim_{x\to \pi} \sec x = \frac{1}{\cos \pi} = \frac{1}{-1} = -1$

(109) $\lim_{x\to \pi} \left[(\cos^2 x - 1)(\csc^2 x)\right] = (1-1)\left(\frac{1}{0}\right) = (0)(\infty)$ Indeterminate

$\lim_{x\to \pi} (-\sin^2 x)\left(\frac{1}{\sin^2 x}\right) = \lim_{x\to \pi} -1 = -1$

(110) $\lim_{x\to \pi} \frac{\tan \frac{x}{2}}{\cot(-x)} \rightarrow \frac{0}{0} = \frac{\frac{1-\cos x}{\sin x}}{\frac{-\cos x}{\sin x}} = \frac{1-\cos x}{\cancel{\sin x}} \cdot \frac{-\cancel{\sin x}}{\cos x} = \frac{-(1-\cos x)}{\cos x} = \frac{1-(-1)}{-1} = -2$

(111) $\lim_{x\to 0} \frac{\csc 8x}{\csc 4x} \to \frac{\infty}{\infty} = \lim_{x\to 0} \frac{\sin 4x}{\sin 8x} = \lim_{x\to 0} \frac{\frac{\sin 4x}{x}}{\frac{\sin 8x}{x}} = \frac{4 \lim_{x\to 0} \frac{\sin 4x}{4x}}{8 \lim_{x\to 0} \frac{\sin 8x}{8x}} = \frac{(4)(1)}{(8)(1)}$

$= \frac{1}{2}$

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(112) $\lim_{x\to \pi/2^-} (\tan x - \sec x) = \infty - \infty \to$ Indeterminate

$\lim_{x\to \pi/2^-} \left(\frac{\sin x}{\cos x} - \frac{1}{\cos x}\right) = \lim_{x\to \pi/2^-} \frac{\sin x - 1}{\cos x} \cdot \left(\frac{\sin x + 1}{\sin x + 1}\right) = \lim_{x\to \pi/2^-} \frac{\sin^2 x - 1}{\cos x(\sin x + 1)} = \lim_{x\to \pi/2^-} \frac{-\cos^2 x}{\cos x(\sin x + 1)}$

$\lim_{x\to \pi/2^-} \frac{-\cos x}{\sin x + 1} = \frac{0}{1+1} = 0$

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(113) $\lim_{x\to \pi/16} (\csc 4x)(\cot \frac{8x}{3} - \frac{\pi}{3}) = (\csc \frac{\pi}{4})(\cot(\frac{\pi}{6} - \frac{\pi}{3})) = (\csc \frac{\pi}{4})(\cot -\frac{\pi}{6})$

$(\sqrt{2})(-\sqrt{3}) = -\sqrt{6}$

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(114) $\lim_{x\to 0} \frac{\sin^2 x}{1 - \sec^2 x} = \frac{0}{1-1} = \frac{0}{0} \to$ Indeterminate

$\lim_{x\to 0} \frac{\sin^2 x}{-\tan^2 x} = \lim_{x\to 0} \frac{\sin^2 x}{\frac{-\sin^2 x}{\cos^2 x}} = \lim_{x\to 0} \cancel{\sin^2 x} \cdot \frac{\cos^2 x}{-\cancel{\sin^2 x}} = \lim_{x\to 0} -\cos^2 x = -1$

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(115) $\lim_{x\to 0} \left(\frac{\sin^2 x + \cos x - 1}{x}\right) \to \frac{0}{0} = \lim_{x\to 0} \frac{\sin^2 x}{x} + \frac{\cos x - 1}{x} = \lim_{x\to 0} \left(\frac{\sin x}{x}\right)(\sin x) + \left(\frac{\cos x - 1}{x}\right)$

$(1)(0) + (0) = 0$

or

$\lim_{x\to 0} \frac{\cancel{1} - \cos^2 x + \cos x - \cancel{1}}{x} = \lim_{x\to 0} \cos x \left[\frac{(1 - \cos x)}{x}\right]$

$= (1)(0) = 0$

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(116) $\lim_{x\to 0} \left(\frac{1 - \cos^2 x}{x}\right) \to \frac{0}{0} = \lim_{x\to 0} \left(\frac{1 - \cos x}{x}\right)\left(\frac{1 + \cos x}{1}\right) = (0)(2) = 0$

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(117) $\lim_{x\to 0} \frac{\sin x + \cos x}{x^2} = \frac{1}{0} \Rightarrow \frac{+}{+} = \infty$

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(118) $\lim_{x\to \pi/4} \frac{\sin^2 x - \cos^2 x}{\tan^2 x - 1} \to \frac{0}{0} = \lim_{x\to \pi/4} \frac{\sin^2 x - \cos^2 x}{\frac{\sin^2 x - \cos^2 x}{\cos^2 x}} = \lim_{x\to \pi/4} \frac{\cancel{\sin^2 x - \cos^2 x}}{1} \cdot \frac{\cos^2 x}{\cancel{\sin^2 x - \cos^2 x}}$

$\lim_{x\to \pi/4} \cos^2 x = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$