## Honors Precalculus/Trigonometry Summer Assignment

Dear HPC Student:
You are receiving this packet because you have enrolled in Honors Precalculus/Trigonometry for the 2016 - 2017 school year. This packet contains materials that you have already learned in your previous math classes. It covers all of the prerequisite skills with which you should be familiar in order to be successful in this class. I am not asking you to learn anything new!

You must complete the packet by the end of the summer. I will be looking at your assignment for completion on the first day of school. You will have a test on this material at the end of the first week or beginning of the second week of school. That exact date will be given to you on the first day of school. We will not be doing any of these exact problems in class, but if you are struggling with any of them, please make time to see me when school starts again. I expect you to have attempted every problem even if you struggle. When working with me, I also expect that you have educated questions to ask that result from trying the problem but getting stuck on a specific step. I am providing you with online resources on our website in an attempt to help you through any confusion you might encounter. Please utilize those resources first. Helpful videos can be accessed on my website http://sosanko.weebly.com under the Honors Precalculus tab. You might need to refer to your Algebra II notebook or other online resources like Khan Academy. Please remember to check my website for valuable resources!!!

Enjoy your summer, and I look forward to working with you in the fall.
Sincerely,

Ms. Sosanko<br>msosanko@wjhsd.net



## Class Requirements:

Please complete the following before the first day of school
Download the remind app and join the HPC Class
I use an app called Remind to send students important messages about class. These might be reminders on upcoming exams or announcements about materials I have uploaded to our website. You are also able to use the app to send me messages if you are working at home and are stuck on a problem.

The app is available for both iphone and android from their respective app stores. The icon looks like this $\rightarrow$


Once you download the app, you need to join our HPC class.
To join our class: text @hpctj to the number 81010. If 81010 does not work, try texting the same code (with the "at" sign) to 412-436-5552.

At no time will I see your phone number nor will you see mine. That is why we are using the app. It is important that only educational messages are sent, or I will be forced to remove you from the online class.

I have placed a link to these instructions on my website under the HPC tab.

I will send a test message sometime during the summer.

## Supplies:

We have a classroom set of TI-83 Plus calculators for you to use in class. We do not use calculators very often, but if you are going to purchase one for home use, I would suggest the TI-84 Plus or the TI-84 Plus Silver Edition. There are other versions of these calculators that add a full-color display as well (TI-84 Plus C Silver Edition and the TI84 Plus CE). If you are going to buy a calculator (which I do not require), go to the College Board website and make sure it is SAT/ACT approved.


Please purchase a 3-ring binder. If you want to use one for the whole year, you will probably need a 3 ". If you want to use a different one for each semester, then $1.5 "$ to $2 "$ should work. I give guided note packets for each section, and so a regular spiral notebook will not suffice. I would rather you not use a folder.


Organization is important, and you may want to refer to these materials when you are taking AP Calculus.

## Cell Phone Policy:

Beginning this school year, all of my classes must adhere to a strict cell phone policy. You will receive a copy of it on the first day of school; however, I do have storage for cell phones. If you do not feel comfortable storing your phone with me, please do not take it out during class at all. If you are caught with a phone during any quiz or test at any time, you will receive a zero on that exam. This is solely to protect test security and is comparable to any standardized testing procedure.
(ACT/SAT/Keystones)

## Rationalizing Denominators

$$
\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}
$$


conjugate

Simplify each of the following by completely rationalizing the denominator (no radicals should be left in the denominators of your final answers). Use conjugates when necessary.

1. $\frac{2}{\sqrt{3}}$
2. $\frac{15}{\sqrt{5}}$
3. $\frac{4}{2 \sqrt{7}}$
4. $\frac{2}{1-\sqrt{5}}$
5. $\frac{4}{\sqrt{2}-3}$
6. $\frac{9-2 \sqrt{3}}{\sqrt{3}+6}$

## Writing Equations of Lines

Write the equation of each line below in slope-intercept form. $\quad y=m x+b$
7. a line with a slope of $\frac{1}{2}$ and a y-intercept of -5
8. a line with a slope of 7 that passes through the points $(4,5)$
9. a line with a slope of 0 that passes through the point $(7,1)$
10. a line with an undefined slope that passes through the point $(8,-1)$
11. a line that passes through the points $(6,-4)$ and $(-2,2)$
12. a line that passes through the points $(5,7)$ and $(5,-6)$

## Function Notation/Evaluating Composite Functions

Perform each operation if $f(x)=3 x-6, g(x)=x^{2}+3$, and $h(x)=-2+x$
13. $f(20)$
14. $g(a-4)$
15. $g(f(-1)$
16. $f(x) \cdot g(x) \cdot h(x)$
17. $\frac{f(x)}{h(x)}$
18. $f(g(x))$

## Graphing Quadratic Functions

Complete the following tables and graph the quadratic function. Reminder, the equation of the axis of symmetry is $x=\frac{-b}{2 a}$.
19. $f(x)=x^{2}+2 x-3$

| opens up or down? |  |
| :---: | :--- |
| $y$-intercept |  |
| Axis of Symmetry |  |
| Vertex | $($ |
| Domain |  |
| Range |  |


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20. $f(x)=-2 x^{2}+8 x-5$

| opens up or down? |  |
| :---: | :--- |
| y-intercept |  |
| Axis of Symmetry |  |
| Vertex | $($ |
| Domain |  |
| Range |  |



## Quadratic Word Problem

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

21. Mr. Goodman is going to fire a rocket to start of this year's Battle of the Classes. He has done testing on the rocket to ensure that it will not hit any students. If the rocket is launched upwards with an initial velocity of 200 feet per second, its height (h) in feet can be found by the function $h(t)=-16 t^{2}+120 t$, where $t$ is the number of seconds since it was launched.
a) How long will it take for the rocket to reach its highest point?
b) What is the maximum height that the rocket will reach?
c) How long will it take for the rocket to hit the ground?

## Rules of Exponents

$\begin{cases}x^{a} \cdot x^{b}=x^{a+b} & x^{0}=1, \quad x \neq 0 \\ \frac{x^{a}}{x^{b}}=x^{a-b} & \left(x^{a}\right)^{b}=x^{a b} \\ x^{-a}=\frac{1}{x^{a}} & (x y)^{a}=x^{a} y^{a} \\ y^{a} & (x \pm y)^{a} \neq x^{a} \pm y^{a} \\ \end{cases}$

Simplify each of the following. All answers should have only positive exponents.
22. $\left(2 x^{2} y\right)^{0}(3 x y)$
23. $a^{-2} b^{3} a^{3}$
24. $\frac{4^{-5} 4^{7}}{4^{3}}$
25. $(2 x)^{-2}(2 y)^{3}(4 x)$
26. $\frac{\left(a^{-3} b^{2} c\right)^{-2}}{\left(3 a b^{-2} c^{3}\right)^{-1}}$
27. $\frac{\sqrt[3]{a^{3} b^{7} c}}{a^{2} b c^{5}}$

## Factoring

Please note: This is the most important section of this assignment. You must know how to factor all of the following expressions to be successful in the HPC class. I am not asking you to actually solve any of the problems - just factor.

## Guidelines for Factoring:

- Always look for a GCF before doing anything else.
- Consider the number of terms in the polynomial
- Two terms (binomial): Try factoring a difference of perfect squares or a sum or difference of two perfect cubes.
$a^{2}-b^{2}=(a+b)(a-b) \quad$ signs are one positive, and one negative $a^{3}-b^{3}=(a-b)\left(a^{2}+a b+b^{2}\right)$ signs are "same, opposite, always positive" $a^{3}+b^{3}=(a+b)\left(a^{2}-a b+b^{2}\right) \quad$ signs are "same, opposite, always positive"
- Three terms (trinomial): Trinomials of the form $a x^{2}+b x+c$ can be factored into the product of two binomials. If $a=1$, this should be very easy for you. If $a \neq 1$ and it cannot be factored out as a GCF, your Algebra 2 teacher either taught you to "guess and check" or to use the key number method.
- More than three terms: Try factoring by grouping
- Make sure that the polynomial is factored completely.

Factor each completely.
28. $8 a b^{2}-4 a b$
29. $x^{2}+4 x-21$
30. $25 x^{2}+10 x y+y^{2}$
31. $9 c^{2}-49 d^{2}$
32. $3 x^{3}-3 x^{2}-90 x$
33. $3 x^{2}+28 x+32$
34. $x^{3}+5 x^{2}-2 x-10$
35. $6 x^{2}+7 x-3$
36. $16 x^{3} y-81 x y$
37. $x^{4}-4 x^{2}-45$
38. $8 x^{3}+125$
39. $18 a^{2}-31 a b+6 b^{2}$
40. $2 x^{2}+5 x+3$
41. $3 x^{2}+7 x+2$
42. $5 x^{2}-7 x+2$
43. $6 x^{2}-11 x+3$
44. $6 x^{2}-13 x-5$
45. $4 x^{2}-11 x-3$
46. $7 x^{2}+9 x+2$
47. $7 x^{2}-10 x+3$
48. $9 x^{2}-16 y^{2}$
49. $x^{2}-64$
50. $16 a^{4}-81 y^{8}$
51. $27 x^{3}+1$
52. $8 a^{3}-y^{3}$
53. $m^{6}-216$
54. $x^{6}-1$
55. $2 a x+6 x c+b a+3 b c$
56. $3 m y+7 x+7 m+3 x y$
57. $a^{2}-2 a b+a-2 b$
58. $x^{3}+2 x^{2}-x-2$

## Rational Expressions:

Remember, if you want to add or subtract fractions, you must have common denominators. To divide fractions, remember "copy, change, flip."

Some Examples to Remind You:

$$
\begin{aligned}
\frac{5 x-1}{x^{2}-3 x+2}+\frac{3}{2 x-4} & =\frac{5 x-1}{(x-1)(x-2)}+\frac{3}{2(x-2)}= \\
& =\frac{2(5 x-1)}{2(x-1)(x-2)}+\frac{3(x-1)}{2(x-1)(x-2)}= \\
& =\frac{2(5 x-1)+3(x-1)}{2(x-1)(x-2)}= \\
& =\frac{13 x-5}{2(x-1)(x-2)}
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{x^{2}+5 x+1}{x+3}-\frac{4 x-5}{x+3}+\frac{7 x+9}{x+3} \\
& =\frac{x^{2}+5 x+1-(4 x-5)+7 x+9}{x+3} \\
& =\frac{x^{2}+5 x+1-4 x+5+7 x+9}{x+3} \\
& =\frac{x^{2}+8 x+15}{x+3} \\
& =\frac{(x+3)(x+5)}{x+3}
\end{aligned}
$$

## Rational Expressions

When dividing rational expressions...

1) Keep, Change, Flip
2) Factor all the numerators and denominators
3) Divide out common factors

$$
\begin{aligned}
& \frac{3 x^{2}+9 x}{x^{2}+5 x+6} \div \frac{x^{2}-9}{x^{2}-x-6} \\
& \frac{3 x^{2}+9 x}{x^{2}+5 x+6} \cdot \frac{x^{2}-x-6}{x^{2}-9} \quad \begin{array}{l}
\text { Keep the first fraction } \\
\text { Flip the e sivisond to multiplication fraction (reciprocal) }
\end{array}
\end{aligned}
$$

$$
\frac{3 x(x \neq 3)}{(x+2)(x+3)} \cdot \frac{(x-3)(x \neq 2)}{(x-3)(x+3)}=\frac{3 x}{x+3}
$$

Simplify each of the following completely by adding, subtracting, multiplying, or dividing.
59. $\frac{x}{x y^{2}}-\frac{2 x}{x^{2}}$
60. $\frac{x}{x-3}+\frac{2}{3 x+4}$
61. $\frac{x^{2}-5 x+6}{x-2}$
62. $\frac{1-x}{x-1}$
63. $\frac{2 x^{2}+x-6}{x^{2}+4 x-5} \cdot \frac{x^{3}-3 x^{2}+2 x}{4 x^{2}-6 x}$
64. $\frac{\frac{x^{2}}{\frac{x-1}{2 x}}}{\frac{2 x}{x-1}}$
65. $\frac{\frac{3}{x+1}-4}{\frac{2 x}{x+1}}$
66. $\frac{\frac{x^{2}+2 x+1}{x^{2}-4}}{\frac{x+1}{x^{2}-x-6}}$
67. $\frac{1+\frac{y}{x}}{\frac{1}{y}+\frac{1}{x}}$
68. $\frac{4}{3 x+6}-\frac{x+1}{x^{2}-4}$
69. $\frac{2 x^{2}-x-15}{x^{2}-13 x+30}$
70. $\frac{\frac{2 x-14}{8 x}}{\frac{x^{2}-49}{4 x}}$

## Basic Polynomial Graphs - Translations/Transformations of Parent Functions

Be able to quickly sketch these basic parent functions from memory:

$$
\begin{gathered}
y=x \quad y=x^{2} \quad y=x^{3} \quad y=|x| \quad y=\sqrt{x} \quad y=\sqrt[3]{x} \quad y=\frac{1}{x} \\
y=\log _{b} x
\end{gathered} y=b^{x} \quad .
$$

Know their domain and range (in interval notation) and whether they are even or odd. Do not worry about end-behavior right now.


Know how to translate or transform each parent function using the rules below.

- To translate or move a parent function, use these basic examples:
- The equation $y=(x-5)^{2}-3$ moves the graph right 5 units and down 3 units
- The equation $y=(x+2)^{2}+4$ moves the graph left 2 units and up 4 units
- To transform or stretch/shrink a parent function, use these basic examples:
- The equation $y=\frac{1}{3} x^{2}$ shrinks the graph vertically. The graph will look wider from left to right.
- The equation $y=4 x^{2}$ stretches the graph vertically. The graph will look thinner from left to right.

Accurately sketch each of the following functions without using a calculator. Include asymptotes if necessary.
71. $y=|x-3|+2$

72. $y=\sqrt{x+2}$

73. $y=\frac{1}{x+2}-4$

75. $y=3(x-1)^{3}$

78. $y=2^{x}-3$

74. $y=\frac{1}{2} x^{2}$

76. $y=\log _{4}(x+2)$

79. $y=\frac{1}{2}|x-4|+5$


## Logarithms

## Rules/Properties to Remember

Left to Right: Condense; Right to Left: Expand
$\log _{b} x+\log _{b} y=\log _{b}(x y) \quad \log _{b} b^{x}=x$
Left to Right: Condense; Right to Left: Expand

$$
\log _{b} x-\log _{b} y=\log _{b}\left(\frac{x}{y}\right) \quad \log _{b} 1=0 \quad \begin{gathered}
\text { because any number to } \\
\text { the } 0 \text { power is } 1
\end{gathered}
$$

$a \log _{b} x=\log _{b} x^{a}$
$\log _{e} x=\ln x($ natural $\log )$
$\log _{b} x=y \quad \leftrightarrow \quad b^{y}=x$
$\log _{10} x=\log x($ common log)
(This process is called exponentiating)

$$
\ln e=1
$$

$\log 10=1$

## Logarithmic Form Exponential Form



Write each of the following logs in exponential form (exponentiate).
80. $\log _{a} x=7$
81. $\ln x=4$
82. $\log y=5$
83. $\log _{\frac{1}{2}} 8=-3$

Write each of the following exponential expressions as logarithms.
84. $e^{5}=b$
85. $10^{x}=y$
86. $2^{3}=8$
87. $\left(\frac{1}{6}\right)^{-1}=6$

Condense each of the following into a single logarithmic expression. All coefficients should be written as exponents. Simplify completely (hint: you might have to factor).
88. $\log x-\log y+4 \log z$
89. $\ln a-\frac{1}{4} \ln b-\ln c^{-3}$
90. $\log _{5}\left(x^{2}-81\right)-\log _{5} 25-\log _{5}\left(x^{2}-7 x-18\right)$
91. $\log \frac{2}{x}+3 \log x^{2}-\frac{1}{2} \log x$
92. $\frac{1}{3} \log _{2}(3 x-6)-3\left[\log _{2}(x+2)-\log _{2} x\right]$
93. $4 \log _{3}(2 x)-\frac{1}{2} \log _{3}\left(4 x^{4}\right)+3 \log _{3} y^{-5}$

Expand each of the following logarithmic expressions completely. Write all exponents as coeffieients. Simplify if necessary.
94. $\ln \left(e^{2} x\right)$
95. $\log _{5}\left[\frac{(x-3)^{2}}{2 \sqrt{x}}\right]^{-3}$
96. $\log _{2}\left(\frac{a b^{2} d^{5}}{c^{12}}\right)^{1 / 4}$
97. $\ln \left[\frac{x^{3}-2 x-3}{(x+1)^{4}}\right]^{5}$
98. $\log \frac{10}{x-y}$
99. $\log _{2} \frac{5 x^{2}}{\left(a^{3} y\right)^{3}}$

Evaluate all of the following logarithmic expressions without using a calculator.
(Hint: $\sqrt[r]{x^{p}}=(\sqrt[r]{x})^{p}=x^{p / r}$ )
100. $\log _{2} 8$
101. $\log 10^{3}$
102. $\ln e^{18}$
103. $14 \ln \sqrt{e}$
104. $\log _{\frac{1}{3}} 3$
105. $\log _{7} \sqrt[3]{7^{6}}$
106. $\log _{\frac{2}{3}} \frac{27}{8}$
108. $\log 2.5-\log 0.025$
110. $\log _{9} 27 \sqrt{3}$

## FractionalExponents

You need to be able to switch between radicals and fractional exponents. For example, if you are given $\sqrt{x^{5}}$, you need to know that this is also $x^{5 / 2}$.

Write each of the following radical expressions using fractional exponents.
112. $\sqrt[3]{x^{7}}$
113. $\sqrt{y^{15}}$
114. $(\sqrt{x})^{19}$
115. $\sqrt[6]{y}$

## More Fractional Exponents

The term under the radical, the radicand, can be a number as well. You need to be able to calculate the value of a number to a fractional exponent (or under a radical) without a calculator.

Example: $8^{2 / 3}$ I can rewrite this as $(\sqrt[3]{8})^{2}$ or as $\sqrt[3]{8^{2}}$. It is typically easier to evaluate the root first because that will make your expression smaller, and then take it to the required power (typically making it larger). That means that the first way I wrote the expression will be easier to evaluate. So $8^{2 / 3}=(\sqrt[3]{8})^{2}=2^{2}=4$.

## Evaluate each of the following without a calculator.

116. $27^{2 / 3}$
117. $125^{-1 / 3}$
118. $2^{10 / 5}$
119. $\left(\frac{8}{25}\right)^{1 / 3}$
120. $\sqrt[3]{125^{-2}}$
121. $32^{2 / 5}$
122. $49^{-.5}$
123. $\sqrt[4]{81^{3}}$
124. $\sqrt[6]{64^{5}}$

## Right Triangle Trigonometry Review (from Geometry)

Recall:

$$
\sin x=\frac{\text { opposite side }}{\text { hypotenuse }} \quad \cos x=\frac{\text { adjacent side }}{\text { hypotenuse }} \quad \tan x=\frac{\text { opposite side }}{\text { adjacent side }}
$$

In the above expressions, $x$ is an angle. Many texts also use $\alpha$ (Greek letter alpha), $\beta$ (Greek letter beta), or $\theta$ (Greek letter theta - most common) to represent angles.

If you are given a drawing of a right triangle, you should be able to find the sine, cosine, and tangent of a specified angle using the ratios as defined by SOH-CAHTOA. There are 3 more ratios that you will be learning in Precalculus/Trigonometry (secant, cosecant, and cotangent). Those are not important right now. Just make sure you know sine, cosine, and tangent ratios.


$$
\begin{array}{lll}
\text { Sine }=\frac{\text { Opposite }}{\text { Hypotenuse }} & \sin x=\frac{3}{5} & \sin y=\frac{4}{5} \\
\text { Cosine }=\frac{\text { Adjacent }}{\text { Hypotenuse }} & \cos x=\frac{4}{5} & \cos y=\frac{3}{5} \\
\text { Tangent }=\frac{\text { Opposite }}{\text { Adjacent }} & \tan x=\frac{3}{4} & \tan y=\frac{4}{3}
\end{array}
$$

You should be able to use these ratios and your scientific calculator to find missing sides and angles of right triangles.

Example: Find a missing angle of a triangle using SOH-CAH-TOA.


Example: Find a missing side of a triangle using SOH-CAH-TOA. Here you are looking for the hypotenuse.

Solve upside down.

$\cos \left(50^{\circ}\right)=\frac{\text { ADJ }}{\text { HYP }} \quad$ Trick: $\quad \mathrm{SOHCAHTOA}$
$\cos \left(50^{\circ}\right)=\frac{25}{x} \Rightarrow \begin{gathered}\text { Switch your } \\ x \text { around! }\end{gathered}$
$x=\frac{25}{\cos \left(50^{\circ}\right)}=\frac{25}{6428}=38.89$

Find the missing side (labeled by the variable) in each diagram below. Round to the nearest tenth. Make sure that your calculator is in degree mode.
126.
127.

128.


Find the missing angle (?) in each diagram below. Round your answer to the nearest degree. Make sure your calculator is in degree mode.
129.



When you are given two sides of a right triangle, you can simply use the Pythagorean Theorem to find the missing third side knowing that $a^{2}+b^{2}=c^{2}$ Where $a$ and $b$ are legs of the right triangle and $c$ is the hypotenuse.



13

$$
\begin{aligned}
12^{2}+y^{2} & =13^{2} \\
144+y^{2} & =169 \\
y^{2} & =25 \\
\sqrt{y^{2}} & =\sqrt{25} \\
y & =5
\end{aligned}
$$

At this point, you might want to do some research on a the topic of Pythagorean Triples. Some helpful ones to know are:

3-4-5
5-12-13
7-24-25

You must have a good grasp of the special right triangles that you learned in Geometry to understand our first topic - the Unit Circle.

The special right triangles that you learned are the 45-45-90 and the 30-60-90. If you know the relationships between the legs and the hypotenuse, then you do not have to use $\mathrm{SOH}-\mathrm{CAH}-\mathrm{TOA}$ or a calculator to find any missing sides.

Study the triangles below and learn how their sides are related.

## $45^{\circ}-45^{\circ}-90^{\circ}$ Triangle Theorem

hypotenuse $=\operatorname{leg} \cdot \sqrt{2}$

$30^{\circ}-60^{\circ}-90^{\circ}$ Triangle Theorem
hypotenuse $=2 \cdot$ shorter leg longer leg $=$ shorter leg $\cdot \sqrt{3}$


Find the missing sides in the triangles below without the use of a calculator. All answers must be completely simplified and rationalized (no radicals in the denominator). The answers must also be exact (no decimals).
131.
132.


2
133.

134.

135.

$y$
136.


Special Note: We are going to be working almost exclusively with a 45-45-90 and a 30-60-90 that have a hypotenuse of 1 . Please "memorize" what they look like.


The information above will allow us to create a very important tool that we will use in Precalculus/Trigonometry called the Unit Circle.

Use the picture below and SOH-CAH-TOA to find the requested information.

146. What do you notice about the sine of $45^{\circ}$ and the cosine of $45^{\circ}$ ?
147. What do you notice about the sine of $30^{\circ}$ and the cosine of $60^{\circ}$ ?
148. What do you notice about the sine of $60^{\circ}$ and the cosine of $30^{\circ}$ ?
149. Can you write a rule about the sine and cosine of "complementary angles" using this information?
150. Write a rule for the relationship between the sine, cosine, and tangent of an angle.
-- Congratulations! You have finished your summer assignment. I look forward to working with you this year!! --

