

$$\textcircled{1} \quad 2\sin x + \sqrt{3} = 0$$

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$x = \frac{4\pi}{3}, \quad x = \frac{5\pi}{3}$$

$$\textcircled{2} \quad \cos(2x) + 6\sin^2 x = 4$$

$$1 - 2\sin^2 x + 6\sin^2 x = 4$$

$$4\sin^2 x = 3$$

$$\sin^2 x = \frac{3}{4}$$

$$\sin x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$\textcircled{3} \quad \cos(2x) = \cos x$$

$$2\cos^2 x - 1 = \cos x$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$$\cos x = -\frac{1}{2}, \quad \cos x = 1$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0\pi$$

$$\textcircled{4} \quad \sin(2x) = \cos x$$

$$2\sin x \cos x - \cos x = 0$$

$$\cos x (2\sin x - 1) = 0$$

$$\cos x = 0, \quad \sin x = \frac{1}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\textcircled{5} \quad \cos(2x) = -\frac{1}{2}$$

$$2x = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$2x = \frac{2\pi}{3} + 2k\pi$$

$$2x = \frac{4\pi}{3} + 2k\pi$$

$$x = \frac{\pi}{3} + k\pi$$

$$x = \frac{2\pi}{3} + k\pi$$

$$x = \frac{\pi}{3}, \quad x = \frac{2\pi}{3}$$

$$x = \frac{4\pi}{3}, \quad x = \frac{5\pi}{3}$$

$$\textcircled{6} \quad 3 - \sin x = \cos(2x)$$

$$3 - \sin x = 1 - 2\sin^2 x$$

$$2\sin^2 x - \sin x + 2 = 0$$

$$\frac{1 \pm \sqrt{1^2 - 4(2)(2)}}{2(2)}$$

$$\frac{1 \pm \sqrt{1-16}}{2}$$

N.S.

$$\textcircled{7} \quad \sin(2x) + 2\sin x \cos(2x) = 0$$

$$2\sin x \cos x + 2\sin x (1 - 2\sin^2 x) = 0$$

$$2\sin x \cos x + 2\sin x - 4\sin^3 x = 0$$

$$2\sin x (\cos x + 1 - 2\sin^2 x) = 0$$

$$2\sin x = 0$$

$$\sin x = 0$$

$$x = 0\pi, \pi$$

$$\cos x + 1 - 2(1 - \cos^2 x) = 0$$

$$\cos x + 1 - 2 + 2\cos^2 x = 0$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\cos x = -1$$

$$x = \pi$$

$$) 2 \cot x - \frac{4}{\cot x} = 7$$

$$2 \cot^2 x - 4 = 7 \cot x$$

$$2 \cot^2 x - 7 \cot x - 4 = 0$$

$$(2 \cot x + 1)(\cot x - 4) = 0$$

$$\cot x = -1/2$$

$$\tan x = -2$$

$$x = \tan^{-1}(-2)$$

$$x = -1.107$$

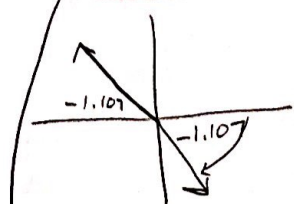
$$\cot x = 4$$

$$\tan x = 1/4$$

$$x = \tan^{-1}(1/4)$$

$$x = .25$$

$$x = 3.38$$



$$x = 2.03$$

$$2\pi - 1.107$$

$$= 5.18$$

$$(10) \sin x \tan x = \sin x$$

$$\sin x \tan x - \sin x = 0$$

$$\sin x (\tan x - 1) = 0$$

$$\sin x = 0$$

$$0\pi, \pi$$

$$\tan x = 1$$

$$\pi/4$$

$$5\pi/4$$

$$(11) \tan^2 x + \tan x - 2 = 0$$

$$(\tan x + 2)(\tan x - 1) = 0$$

$$\tan x = -2$$

$$x = \tan^{-1}(-2)$$

$$x = -1.107$$

$$\rightarrow 5.18$$

$$x = 2.03$$

$$\tan x = 1$$

$$x = \pi/4$$

$$5\pi/4$$

$$(9) \tan(3x) = -\sqrt{3}$$

$$3x = \tan^{-1}(-\sqrt{3})$$

$$3x = \frac{2\pi}{3} + k\pi$$

$$3x = \frac{5\pi}{3} + k\pi$$

$$x = \frac{2\pi}{9} + \frac{k\pi}{3}$$

$$x = \frac{5\pi}{9} + \frac{k\pi}{3}$$

$$x = \frac{2\pi}{9} \quad x = \frac{8\pi}{9}$$

$$x = \frac{5\pi}{9} \quad x = \frac{11\pi}{9}$$

$$x = \frac{5\pi}{9}$$

$$x = \frac{11\pi}{9}$$

$$x = \frac{17\pi}{9}$$

$$x = \frac{8\pi}{9}$$

$$x = \frac{14\pi}{9}$$

$$1) 2 \sin\left(\frac{x}{2}\right) = 1$$

$$\sin\left(\frac{x}{2}\right) = \frac{1}{2}$$

$$\frac{x}{2} = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\frac{x}{2} = \frac{\pi}{6} + 2k\pi$$

$$x = \frac{\pi}{3} + 4k\pi$$

$$x = \frac{\pi}{3}$$

$$\frac{x}{2} = \frac{5\pi}{6} + 2k\pi$$

$$x = \frac{5\pi}{3} + 4k\pi$$

$$x = \frac{5\pi}{3}$$

$$(13) \tan\left(x - \frac{\pi}{2}\right) = 1$$

$$x - \frac{\pi}{2} = \frac{\pi}{4} + k\pi$$

$$x = \frac{\pi}{4} + \frac{\pi}{2} + k\pi$$

$$x = \frac{3\pi}{4} + k\pi$$

$$x - \frac{\pi}{2} = \frac{5\pi}{4} + k\pi$$

$$x = \frac{\pi}{2} + \frac{5\pi}{4} + k\pi$$

$$x = \frac{7\pi}{4} + k\pi$$

$$x = \frac{7\pi}{4}$$

$$x = \frac{11\pi}{4}$$

$$x = \frac{3\pi}{4}$$
$$x = \frac{7\pi}{4}$$
$$x = \frac{11\pi}{4}$$

$$(14) \sin\left(3x + \frac{\pi}{18}\right) = 1$$

$$3x + \frac{\pi}{18} = \frac{\pi}{2} + 2k\pi$$

$$3x = \frac{\pi}{2} - \frac{\pi}{18} + 2k\pi$$

$$3x = \frac{4\pi}{9} + 2k\pi$$

$$x = \frac{4\pi}{27} + \frac{2k\pi}{3}$$

$$x = \frac{4\pi}{27}$$

$$x = \frac{4\pi}{27} + \frac{2\pi}{3} = \frac{22\pi}{27}$$

$$x = \frac{4\pi}{27} + \frac{4\pi}{3} = \frac{40\pi}{27}$$

$$x = \frac{4\pi}{27} + 2\pi = 4\pi \text{ B.S.}$$

$$(15) \cos x = \sec x$$

$$\cos x = \frac{1}{\cos x}$$

$$\cos^2 x = 1$$

$$\cos x = \pm 1$$

$$x = (0\pi, \pi)$$

$$(16) \csc^2 x = 2 \csc x$$

$$\csc^2 x - 2 \csc x = 0$$

$$\csc x (\csc x - 2) = 0$$

$$\csc x = 0 \quad \csc x = 2$$

$$\sin x = \emptyset$$

N.S.

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$(17) 1 + \sin x = 2 \cos^2 x$$

$$1 + \sin x = 2(1 - \sin^2 x)$$

$$1 + \sin x = 2 - 2 \sin^2 x$$

$$2 \sin^2 x + \sin x - 1 = 0$$

$$(2 \sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2}$$

$$\sin x = -1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \frac{3\pi}{2}$$

$$(18) 2 \cos^2 x - \sqrt{3} \cos x = 0$$

$$\cos x (2 \cos x - \sqrt{3}) = 0$$

$$\cos x = 0$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{\pi}{6}, \frac{11\pi}{6}$$

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$$\cos x = .4$$

$$\cos^{-1}(.4) = x$$

$$x = 1.16$$

$$x = 5.12$$

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$$\sin x = -1/9$$

$$x = \sin^{-1}(-1/9)$$

$$x = -.11$$

$$x = 6.17$$

$$x = 3.25$$

$$(58) \quad y = \frac{x^3 - 1}{x^2 - 9} = \frac{(x-1)(x^2 + x + 1)}{(x+3)(x-3)}$$

VA $x=3$ $x=-3$ NO HOLES

HA NONE ($N > D$)

OA $x+0 \rightarrow y=x$

$$\begin{array}{r} x^2+0x-9 \overline{) x^3+0x^2+0x-1} \\ \underline{-(x^3+0x^2-9x)} \\ 0x^2+9x-1 \\ \underline{-(0x^2+0x-0)} \\ 9x-1 \end{array}$$

Cross oblique Asymptote?

$$\frac{x^3-1}{x^2-9} = \frac{x}{1}$$

$$x^3 - 9x = x^3 - 1$$

$$-9x = -1$$

$$x = \frac{1}{9}$$

x-int $x^3 - 1 = 0$

$$x^3 = 1$$

$$x = 1$$

y-int $\frac{0^3 - 1}{0^2 - 9} = \frac{1}{9}$

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$$y = \frac{x^4 + x^2 + 1}{x^2 - 1} = \frac{x^4 + x^2 + 1}{(x+1)(x-1)}$$

VA $x=1$ $x=-1$ NO HOLES

HA NONE

OA NONE

x-int NONE $x^4 + x^2 + 1 \neq 0$

y-int $\frac{1}{-1} = -1$

60

$$y = \frac{x^2}{x^2 + x - 6} = \frac{x^2}{(x+3)(x-2)}$$

VA $x=-3$ $x=2$ NO HOLES

HA $y=1$ x-int $(0,0)$

OA NONE y-int $(0,0)$

CROSS HA?

$$\frac{x^2}{x^2 + x - 6} = 1$$

$$x^2 + x - 6 = x^2$$

$$x - 6 = 0$$

$$\boxed{x=6}$$

$$(61) \quad y = \frac{3x}{x^2-1} = \frac{3x}{(x+1)(x-1)}$$

VA $x=1$ $x=-1$

HA $y=0$

OA NONE

CROSS HA?

$$\frac{3x}{x^2-1} = 0$$

$$3x = 0$$

$$x = 0$$

x-int (0,0)

y-int (0,0)

$$(62) \quad y = \frac{2x^2}{x^2+x-2} = \frac{2x^2}{(x+2)(x-1)}$$

VA $x=-2$ $x=1$

HA $y=1$

OA NONE

CROSS HA?

$$\frac{2x^2}{x^2+x-2} = 1$$

$$2x^2 = x^2 + x - 2$$

$$x^2 - x + 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x=2 \quad x=-1$$

x-int (0,0)

y-int (0,0)

$$(63) \quad Y = \frac{5x}{x^2+x-12} = \frac{5x}{(x+4)(x-3)}$$

VA $x = -4$ $x = 3$ no holes

HA $y = 0$ x-int $(0,0)$

GA NONE y-int $(0,0)$

CROSS HA?

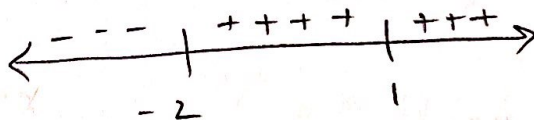
$$\frac{5x}{x^2+x-12} = 0$$

$$5x = 0$$

$$x = 0$$

$$(70) \quad (x-1)^2 (x+2)^3 \geq 0 \rightarrow + \text{ or } 0$$

Zeros $x = 1$
 $x = -2$



$$f(-3) = (-)^2 (-)^3 = (+)(-) = -$$

$$f(0) = (-)^2 (+)^3 = (+)(+) = +$$

$$f(2) = (+)(+) = +$$

$$\boxed{[-2, \infty)}$$

$$(71) \quad 2x^3 - 3x^2 - 32x + 48 > 0 \rightarrow +$$

$$x^2(2x-3) - 16(2x-3) > 0$$

$$(x^2-16)(2x-3) = 0$$

$$x = \pm 4 \quad x = 3/2$$

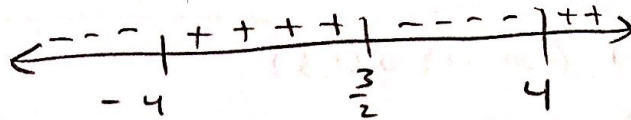
$$(x+4)(x-4)(2x-3)$$

$$f(-5) = (-)(-)(-) = -$$

$$f(0) = (+)(-)(-) = +$$

$$f(2) = (+)(-)(+) = -$$

$$f(5) = (+)(+)(+) = +$$

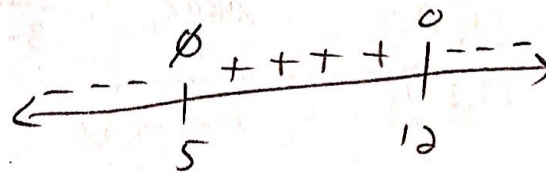


$$(-4, \frac{3}{2}) \cup (4, \infty)$$

$$(72) \quad \frac{x-12}{5-x} \leq 0 \text{ neg or } 0$$

$$\text{zero: } x=12$$

$$\emptyset \quad x=5$$



$$(-\infty, 5) \cup [12, \infty)$$

$$f(0) = \frac{-}{+} = -$$

$$f(6) = \frac{-}{-} = +$$

$$f(13) = \frac{+}{-} = -$$

(73) $x^2 + 9 \leq 0$ Never \emptyset

(81) a) $(-1, 1) \cup (3, \infty)$

b) $(-\infty, -1) \cup (1, 3)$

c) $(1, 2)$

d) $(-1, 1) \cup (3, 0)$

e) NONE

f) $(3, 0)$

(82) Every continuous function on a closed interval has an abs max & abs min on that interval

(83) $f(x) = 3x^2$

$f(1) = 3$ $(1, 3)$

$f(3) = 27$ $(3, 27)$

$$A_{ROC} = \frac{27-3}{3-1} = \frac{24}{2} = 12$$

(84) secant

(85) $f(x) = 3x^2 - 2x + 3$

$$f(-2) = 3(-2)^2 - 2(-2) + 3 \\ = 12 + 4 + 3 = 19$$

$$f(1) = 3 - 2 + 3 = 4$$

$$A_{ROC} = \frac{19-4}{-2-1} = \frac{15}{-3} = -5 = m$$

$$y = mx + b$$

$$4 = -5(1) + b$$

$$4 = -5 + b$$

$$b = 9$$

$$y = -5x + 9$$

86) Instantaneous

$$(87) \quad f(x+h) = (x+h)^2 + 2(x+h) = x^2 + 2xh + h^2 + 2x + 2h$$

$$\frac{x^2 + 2xh + h^2 + 2x + 2h - (x^2 + 2x)}{h} = \frac{2xh + h^2 + 2h}{h}$$

$$= \boxed{2x + h + 2}$$

88) difference

89) derivative

$$(90) \quad \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} = \frac{\frac{-h}{x(x+h)}}{h} = \boxed{\frac{-1}{x(x+h)}}$$

$$(91) \quad \frac{\sqrt{x+h-4} - \sqrt{x-4}}{h} = \frac{\sqrt{x+h-4} + \sqrt{x-4}}{\sqrt{x+h-4} + \sqrt{x-4}}$$

$$= \frac{x+h-4 - (x-4)}{h(\sqrt{x+h-4} + \sqrt{x-4})} = \boxed{\frac{1}{\sqrt{x+h-4} + \sqrt{x-4}}}$$

92)

$$\frac{x^2}{16} + \frac{y^2}{7} = 1$$

$$a = 4 \quad a^2 = 16$$

$$b = \sqrt{7} \quad b^2 = 7$$

$$c = 3 \quad c^2 = 9$$

93)

center (0,0)

$$a = 5 \quad a^2 = 25$$

$$b = 4 \quad b^2 = 16$$

$$c = 3 \quad c^2 = 9$$

major axis \rightarrow x axis

foci (3,0) (-3,0)

vertices (5,0) (-5,0)

y-int (0,4) (0,-4)

94)

$$4x^2 - 8x + \underline{\quad} + y^2 + 4y + \underline{\quad} = -4 + \underline{\quad} + \underline{\quad}$$

$$4(x^2 - 2x + \frac{1}{4}) + y^2 + 4y + \frac{4}{4} = -4 + \frac{4}{4} + \frac{4}{4}$$

$$4(x-1)^2 + (y+2)^2 = 4$$

$$(x-1)^2 + \frac{(y+2)^2}{4} = 1$$

$$a = 2 \quad a^2 = 4 \quad b^2 = a^2 - c^2$$

$$b = 1 \quad b^2 = 1 \quad c^2 = a^2 - b^2$$

$$c = \sqrt{3} \quad c^2 = 3$$

Center (1, -2)

Major axis is parallel to y-axis

vertices (0,2) (0,-2)

foci (0, \sqrt{3}) (0, -\sqrt{3})

x-int (1,0) (-1,0)

(95) center (3, -6)
 $r = 10$

(96) $x^2 - 6x + \frac{9}{4} + y^2 + 14y + \frac{49}{4} = -12 + \frac{9}{4} + \frac{49}{4}$
 $(x-3)^2 + (y+7)^2 = 46$
 center (3, -7)
 $r = \sqrt{46}$

$C = 2\pi r$
 $24\pi = 2\pi r$
 $r = 12$

(97) $(x + \frac{1}{2})^2 + (y-4)^2 = 144$

(98) $x^2 = -4y$ ↓
 vertex (0, 0)
 focus (0, -1)
 directrix $y = 1$
 $LR = 4 \rightarrow 2$ (2, -1) (-2, -1)
 ↖ ↗

99 $(x-1)^2 + (y-2)^2$