Derivative Chapter Review

① False; The derivative is an <u>equation</u> that gives the slope of the tangent line at a point on the graph.

② True [Also know that AROC is the slope of the secant line]

③ False; The formula is $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

④ False; some rational functions are differentiable for all real numbers

$f(x) = \frac{x-3}{x^2+4}$ ← never 0

but some have points of discontinuity

$f(x) = \frac{x-3}{(x+2)(x-1)}$ $x \neq -2$; $x \neq 1$

⑤ A function is non-differentiable at $x=a$ (a point) when the graph of the function:

① is discontinuous at $x=a$ (hole, vertical asymptote)

② has a sharp point or cusp at $x=a$

③ has a vertical tangent line at $x=a$

④ when $x=a$ is an endpoint of the graph

⑥ A derivative is a function that gives the slope of a tangent line at any point on the graph.

$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ the function represents the slope of a line between 2 points that are getting infinitely close together

→ a secant line becoming a tangent line

⑦ $\frac{f(x+h)-f(x)}{h}$ is the difference quotient

it gives the slope of a secant line between $x+h$ and x. This is also the average rate of change.

⑧ AROC $\frac{f(x+h)-f(x)}{h}$ or $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$f(x) = 2x - 3x^2$

$f(-1) = 2(-1) - 3(-1)^2 = -2 - 3(1) = -2 - 3 = -5$ $(-1, -5)$

$f(3) = 2(3) - 3(3)^2 = 6 - 3(9) = 6 - 27 = -21$ $(3, -21)$

$AROC = \frac{-5-(-21)}{-1-3} = \frac{-5+21}{-4} = \frac{16}{-4} = -4$

⑨ IROC $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ → you can use this formula + let $x = 1$ <u>or</u> you can find the derivative (using short cuts) and plug in 1 at the end.

↑ easier!!

$f(x) = \ln x + 3x^2$

$f'(x) = \frac{1}{x} + 6x$

$f'(1) = \frac{1}{1} + 6(1)$

$= 1 + 6 = 7$

⑩ $f(x)$ has a sharp point when $x = -2$

abs val graph

⑪ $x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0$ $x = 2$

$f(x)$ is discontinuous so it's non-differentiable there

(12) $f(x) = \frac{2}{3x} + \frac{x^2}{5} + 2e^3$ → power

$f(x) = \frac{2}{3}x^{-1} + \frac{1}{5}x^2 + 2e^3$

$f'(x) = -\frac{2}{3}x^{-2} + \frac{2}{5}x + 0$

(13) $g(x) = \frac{3}{\sqrt{x^2 - x}}$ → chain

$g(x) = 3(x^2 - x)^{-1/2}$

$g'(x) = -\frac{3}{2}(x^2 - x)^{-3/2}(2x - 1)$

(14) $h(x) = \left(\frac{7x-5}{9+x}\right)^4$ → chain then Quotient

$h'(x) = 4\left(\frac{7x-5}{9+x}\right)^3 \left(\frac{(9+x)(7x-5)' - (7x-5)(9+x)'}{(9+x)^2}\right)$

$= 4\left(\frac{7x-5}{9+x}\right)^3 \left(\frac{(9+x)(7) - (7x-5)(1)}{(9+x)^2}\right)$

(15) $p(x) = \left(\frac{4-x^2}{2x^3 - 6x}\right)\sqrt{7+x^2}$ → product, quotient, chain

$p(x) = \left(\frac{4-x^2}{2x^3-6x}\right)(7+x^2)^{1/2}$

→ chain → Quotient

$p'(x) = \left(\frac{4-x^2}{2x^3-6x}\right)\left[(7+x^2)^{1/2}\right]' + (7+x^2)^{1/2}\left(\frac{4-x^2}{2x^3-6x}\right)'$

$p'(x) = \left(\frac{4-x^2}{2x^3-6x}\right)\left(\frac{1}{2}(7+x^2)^{-1/2}\right)(2x) + (7+x^2)^{1/2}\frac{(2x^3-6x)(-2x) - (4-x^2)(6x^2-6}{(2x^3-6x)^2}$

(16) $f(x) = 2xe^{-x} - x\ln x$ → **PRODUCTS!** 2

$f'(x) = (2x)(-e^{-x}) + (e^{-x})(2) - \left[(x)\left(\frac{1}{x}\right) + (\ln x)(1)\right]$

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(17) $f(x) = \ln \sqrt[4]{x^3} + e^x - x^2$ → NOT A CHAIN RULE!

$f(x) = \ln x^{3/4} + e^x - x^2$

$f(x) = \frac{3}{4}\ln x + e^x - x^2$ → log prop. $\ln x^b = b\ln x$

$f'(x) = \frac{3}{4}\left(\frac{1}{x}\right) + e^x - 2x$

(18) $f(x) = \frac{e^{x^2+1}}{\ln(2x+4)}$ → QUOTIENT

$e^{f(x)}$ $\ln f(x)$

$f'(x) = \frac{(\ln(2x+4))(e^{x^2+1})' - (e^{x^2+1})(\ln(2x+4))'}{(\ln(2x+4))^2}$

$f'(x) = (\ln(2x+4))(e^{x^2+1})(2x) - (e^{x^2+1})\left(\frac{(2x+4)'}{2x+4}\right)$

$= (\ln(2x+4))(e^{x^2+1})(2x) - (e^{x^2+1})\left(\frac{2}{2x+4}\right)$

(19) $f(x) = \ln \sqrt[3]{\sqrt{x} + 3x}$

$f(x) = \ln (x^{1/2} + 3x)^{1/3}$ log prop

$f(x) = \frac{1}{3}\ln(x^{1/2} + 3x)$

$f'(x) = \frac{1}{3}\left(\frac{\frac{1}{2}x^{-1/2} + 3}{x^{1/2} + 3x}\right)$

(20) $f(x) = 7^{2^{\ln x^2}}$ → $\ln x^2 = 2\ln x$

$f(x) = 7^{2^{2\ln x}}$

$f'(x) = (\ln 7)(7^{2^{2\ln x}})(2^{2\ln x})'$

$f'(x) = (\ln 7)(7^{2^{2\ln x}})(\ln 2)(2^{2\ln x})(2\frac{1}{x})$

(21) $f(x) = \dfrac{\sqrt[4]{\ln(x^3+3)}}{4e^{ex+x^e}}$

Note that ln is taken to the power → that means chain

$$f(x) = \frac{\left(\ln(x^3+3)\right)^{1/4}}{4e^{ex+x^e}}$$

→ chain rule } Quotient rule

$$f'(x) = \frac{\left(4e^{ex+x^e}\right)\left(\left(\ln(x^3+3)\right)^{1/4}\right)' - \left(\left(\ln(x^3+3)\right)^{1/4}\right)\left(4e^{ex+x^e}\right)'}{\left(4e^{ex+x^e}\right)^2}$$

$$f'(x) = \frac{\left(4e^{ex+x^e}\right)\left(\frac{1}{4}\left(\ln(x^3+3)\right)^{-3/4}\right)\left(\frac{3x^2}{x^3+3}\right) - \left(\ln(x^3+3)\right)^{1/4}\left(4e^{ex+x^e}\right)\left(e+ex^{e-1}\right)}{\left(4e^{ex+x^e}\right)^2}$$

(22) $f(x) = \left(\dfrac{\ln x}{e^x}\right)^3$ chain, quotient

$$f'(x) = 3\left(\frac{\ln x}{e^x}\right)^2 \frac{\left(e^x\left(\frac{1}{x}\right) - (\ln x)(e^x)\right)}{(e^x)^2}$$

common log is base 10

(23) $f(x) = \log[\ln(e^x+x^2)]$

$$f'(x) = \left(\frac{1}{\ln 10}\right)\left(\frac{\left(\ln(e^x+x^2)\right)'}{\ln(e^x+x^2)}\right)$$

$$f'(x) = \left(\frac{1}{\ln 10}\right)\left(\frac{\frac{(e^x+x^2)'}{e^x+x^2}}{\ln(e^x+x^2)}\right)$$

→ $f'(x) = \dfrac{1}{\ln 10}\left(\dfrac{\frac{e^x+2x}{e^x+x^2}}{\ln(e^{x+x^2})}\right)$

<u>repeated</u> use of $\dfrac{f'(x)}{f(x)}$

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(24) $f(x) = \dfrac{1}{(e^{3-x+4x})^5}$

$f(x) = (e^{3+3x})^{-5}$

$f'(x) = -5(e^{3+3x})^{-6}(e^{3+3x})(3)$

(25) $f(x) = \log 5^{3x^2-7}$ log prop

$f(x) = (3x^2-7)(\log 5)$ ← just a number

$f'(x) = (\log 5)(6x)$

(26) $h(x) = \dfrac{2x-5}{2x-3}$

$h'(x) = \dfrac{(2x-3)(2) - (2x-5)(2)}{(2x-3)^2}$

$h'(2) = \dfrac{(2(2)-3)(2) - (2(2)-5)(2)}{(2(2)-3)^2}$

$= \dfrac{(4-3)(2) - (4-5)(2)}{(4-3)^2}$

$= \dfrac{2-(-2)}{1}$

$= 4$

(27) $k(x) = x(3x-2)^4$

$k'(x) = x(4(3x-2)^3)(3) + (3x-2)^4(1)$

$k'(1) = (1)(4)(1)^3(3) + (1)^4(1)$

$= 12+1 = 13$

(28) $f(x) = 1 + e^x$

$f'(x) = e^x$

$f'(0) = e^0 = 1$

(29) $g(x) = (\ln x)^3$

Not a log prop
chain rule!

$g'(x) = 3(\ln x)^2\left(\frac{1}{x}\right)$

$g'(e) = 3(\ln e)^2\left(\frac{1}{e}\right)$

$= 3(1)^2\left(\frac{1}{e}\right)$

$= \dfrac{3}{e}$