2016 Final Review

1. X-int or pt. of dis. : I.P.
2. definite Integral
3. Extrema
4. $\frac{dx}{dt} = \frac{1}{y}\frac{dy}{dt} \rightarrow \boxed{\frac{dy}{dt} = y\frac{dx}{dt}}$
5. ~~[illegible]~~ Anti-differentiation or Integration
6. I.P.
7. local maximum
8. negative
9. local maximum
10. x-int. or pt. of dis.
11. Riemann Sum
12. local extrema
13. partition # : Domain
14. absolute maximum, absolute minimum
15. concave up: Positive

16. TRUE
17. False
18. TRUE
19. TRUE
20. False
21. False
22. True
23. TRUE
24. False
25. False

26. $x=3$
27. $(1,\infty)$
28. $x=1$
29. $(-1,3)$
30. $(-\infty,-1)\cup(3,\infty)$
31. $x=1$
32. $(-\infty,1)$
33. $(-\infty,-1.8)\cup(-0.1,4.9)$
34. no where
35. $(1,\infty)$
36. no where
37. $x=1$
38. $x=-1,\ x=3$

39. If graph is $f'(x)$

$\boxed{26}$ $x=-1.8$, $x=4.9$

$\boxed{27}$ $(-\infty,-1)\cup(3,\infty)$

$\boxed{28}$ $x=-1,\ x=3$

$\boxed{29}$ $(-\infty,-1.8)\cup(-.1,4.9)$

$\boxed{30}$ $(-1.8,-.1)\cup(4.9,\infty)$

$\boxed{31}$ $x=-1,\ x=3$

$\boxed{32}$ $(-1,3)$

$\boxed{35}$ $(-\infty,-1)\cup(3,\infty)$

$\boxed{36}$ $x=-1$

$\boxed{37}$ $x=-1,\ x=3$

40.

$\boxed{27}$ $(-1.8,-.1)\cup(4.9,\infty)$

$\boxed{28}$ $x=-1.8,\ -.1,\ 4.9$

$\boxed{31}$ $x=-1.8,\ -.1,\ 4.9$

$\boxed{32}$ $(-\infty,-1.8)\cup(-.1,4.9)$

$\boxed{35}$ $(-1.8,-.1)\cup(4.9,\infty)$

$\boxed{37}$ $x=-1.8,\ -.1,\ 4.9$

41. $\int_2^2 f(x)\,dx = 0$

42. $\int \frac{1}{3}x^{-1/2}\,dx$

$\frac{2}{3}x^{1/2}+C$

43. $\int \frac{\ln x}{x}\,dx$ $\quad u=\ln x$

$\int \ln x\left(\frac{1}{x}\right)dx$ $\quad du=\frac{1}{x}dx$

$\int u\,du = \frac{u^2}{2}+C = \frac{1}{2}(\ln x)^2+C$

44. $\int 5x(x^2-4)^6\,dx$ $\quad u=x^2-4$, $du=2x\,dx$

$\frac{5}{2}\int u^6\,du$

$\frac{5}{2}\left(\frac{u^7}{7}\right)+C = \frac{5}{14}(x^2-4)^7+C$

45. $\int x(x-1)\,dx$

$\int(x^2-x)\,dx = \frac{x^3}{3}-\frac{x^2}{2}+C$

46. $\int \frac{x+2}{x^2+4x}\,dx$ $\quad u=x^2+4x$, $du=2x+4\,dx$

$\frac{1}{2}\int\frac{1}{u}\,du$

$\frac{1}{2}\ln|u|+C = \frac{1}{2}\ln|x^2+4x|+C$

(47) $\int \frac{xe^{x}}{e^{x}} - \frac{e^{-2x}}{e^{x}}\,dx = \int (x - e^{-3x})\,dx = \frac{x^2}{2} + \frac{1}{3}e^{-3x} + C$

(48) $\int \left(\frac{1}{2x-1}\right)\left(\frac{1}{\ln(2x-1)}\right)dx$

$u = \ln(2x-1)$

$du = \frac{2}{2x-1}\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|\ln(2x-1)| + C$

(49) $\int 7x\,e^{3x^2}\,dx$

$u = 3x^2$

$du = 6x\,dx$

$7\int x e^{3x^2}\,dx$

$\frac{7}{6}\int e^{u}\,du = \frac{7}{6}e^{u} + C = \frac{7}{6}e^{3x^2} + C$

(50) $\int \frac{2+e^{-2x}}{e^{2x}}\,dx$

$u = 2 + e^{-2x}$

$du = -2e^{-2x}\,dx$

$\int (2+e^{-2x})(e^{-2x})\,dx$

$-\frac{1}{2}\int u\,du = -\frac{1}{2}\left(\frac{u^2}{2}\right) + C = -\frac{1}{4}(2+e^{-2x})^2 + C$

or

$\int \frac{2}{e^{2x}} + \frac{e^{-2x}}{e^{2x}}\,dx$

$\int (2e^{-2x} + e^{-4x})\,dx$

$-e^{-2x} - \frac{1}{4}e^{-4x} + C$

(51) $\int x(3-x^2)^2\,dx$

$u = 3 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}\int u^2\,du$

$-\frac{1}{2}\left(\frac{u^3}{3}\right) + C = -\frac{1}{6}(3-x^2)^3 + C$

or

$\int x(9 - 6x^2 + x^4)\,dx$

$\int (9x - 6x^3 + x^5)\,dx$

$\frac{9x^2}{2} - \frac{3}{2}x^4 + \frac{x^6}{6} + C$

(52) $f(x) = x^4 + 4x^3 + 4x^2 + 1$

$f'(x) = 4x^3 + 12x^2 + 8x$

$f'(x) = 4x(x^2 + 3x + 2)$

$f'(x) = 4x(x+2)(x+1)$

$f'(x)$: $-$ 0 $+$ 0 $-$ 0 $+$ at -2, -1, 0

P.N. & C.V.

$x = 0$

$x = -2$

$x = -1$

Inc: $(-2,-1) \cup (0,\infty)$

Dec: $(-\infty,-2) \cup (-1,0)$

(53) $g(x) = \frac{x+2}{x+1}$

$g'(x) = \frac{(x+1)(1) - (1)(x+2)}{(x+1)^2}$

$g'(x) = \frac{-1}{(x+1)^2}$

$f'(x)$: $-$ ND $-$ at -1

P.N.

$x = -1$

C.V.

None

Inc: None

Dec: $(-\infty,-1) \cup (-1,\infty)$

(54) $f(x) = x - 4\ln(3x-9)$

$f'(x) = 1 - 4\left(\frac{3}{3x-9}\right)$

$f'(x) = \frac{3x-9-12}{3x-9} = \frac{3x-21}{3x-9}$

$f'(x)$: $-$ 0 $+$ at 3, 7

P.N. & C.V.

$3x - 21 = 0$

$3x = 21$

$x = 7$

~~$3x - 9 = 0$~~

~~$3x = 9$~~

~~$x = 3$~~

Domain

$x > 3$

Inc: $(7,\infty)$

Dec: $(3,7)$

(55) $h(x) = x\sqrt{9-x^2} = x(9-x^2)^{1/2}$ Domain: $(-3,3)$

$h'(x) = (x)\left[\frac{1}{2}(9-x^2)^{-1/2}(-2x)\right] + (1)(9-x^2)^{1/2}$

$h'(x) = \frac{-x^2}{\sqrt{9-x^2}} + \sqrt{9-x^2} = \frac{-x^2+9-x^2}{\sqrt{9-x^2}}$

$h'(x) = \frac{-2x^2+9}{\sqrt{9-x^2}}$

$f'(x)$: ND $-$ 0 $+$ 0 $-$ ND at -3, -2.12, 2.12, 3

P.N. & C.V.

$9 - x^2 = 0$

$9 = x^2$

$x = \pm 3$

$-2x^2 + 9 = 0$

$x^2 = \frac{9}{2}$

$x = \pm\sqrt{\frac{9}{2}}$

$x = \pm\frac{3}{\sqrt{2}}$

$x = \pm\frac{3\sqrt{2}}{2}$

$x = \pm 2.12$

Dec: $(-3,-2.12) \cup (2.12,3)$

Inc: $(-2.12, 2.12)$

(56) $p(x) = x^{1/3} + x^{4/3}$

$p'(x) = \frac{1}{3}x^{-2/3} + \frac{4}{3}x^{1/3}$

$p'(x) = \frac{1}{3x^{2/3}} + \frac{4x^{1/3}}{3}$

$p'(x) = \frac{1+4x}{3x^{2/3}}$

P.N. & C.V.

$1 + 4x = 0$

$4x = -1$

$x = -\frac{1}{4}$

$x = 0$

$-$ 0 $+$ ND $+$ at $-\frac{1}{4}$, 0

Dec: $(-\infty,-\frac{1}{4})$

Inc: $(-\frac{1}{4},0) \cup (0,\infty)$

(57) $j(x) = xe^{x^2-3x}$

$j'(x) = (x)(e^{x^2-3x})(2x-3) + (e^{x^2-3x})$

$j'(x) = (e^{x^2-3x})(2x^2-3x+1)$

$j'(x) = (e^{x^2-3x})(2x-1)(x-1)$

P.N. & C.V.

$e^{x^2-3x} = 0$ → N.S.

$2x - 1 = 0$, $x = \frac{1}{2}$

$x - 1 = 0$, $x = 1$

$f'(x)$: $+$ 0 $-$ 0 $+$ at $\frac{1}{2}$, 1

Dec: $(\frac{1}{2},1)$

Inc: $(-\infty,\frac{1}{2}) \cup (1,\infty)$

(58) $h(x) = \ln\left(\frac{5x^2+4}{x^2+1}\right)$

$h(x) = \ln(5x^2+4) - \ln(x^2+1)$

$h'(x) = \frac{10x}{5x^2+4} - \frac{2x}{(x^2+1)} = \frac{10x^3+10x-10x^3-8x}{(5x^2+4)(x^2+1)}$

$h'(x) = \frac{2x}{(5x^2+4)(x^2+1)}$

P.N. & C.V.

$x = 0$

$f'(x)$: $-$ 0 $+$ at 0

Dec: $(-\infty,0)$

Inc: $(0,\infty)$

(59) $f(x) = 1 + 2x^{-1} - 4x^{-2}$

$f'(x) = 0 - 2x^{-2} + 8x^{-3}$

$f'(x) = \frac{-2}{x^2} + \frac{8}{x^3}$

$f'(x) = \frac{-2x+8}{x^3}$

$f'(x)$: $-$ ND $+$ 0 $-$ at 0, 4

P.N.

$-2x + 8 = 0$

$8 = 2x$

$x = 4$

$x = 0$

C.V.

$x = 4$

Dec: $(-\infty,0) \cup (4,\infty)$

Inc: $(0,4)$

(60) $f(x) = \frac{x-2}{(x^2)}$

$f'(x) = \frac{(x^2)(1) - (2x)(x-2)}{(x^2)^2}$

$f'(x) = \frac{x^2-2x^2+4x}{x^4} = \frac{4x-x^2}{x^4}$

$f'(x) = \frac{4-x}{x^3}$

P.N.

$x = 4$

$x = 0$

C.V.

$x = 4$

$-$ ND $+$ 0 $-$ at 0, 4

$x = 4$ local max

(61) $h(x) = 2x + \ln x$ $x > 0$

$h'(x) = 2 + \frac{1}{x}$

$h'(x) = \frac{2x+1}{x}$

$+$ on $(0, \infty)$

PN & C.V.

$2x + 1 = 0$

~~$x = -\frac{1}{2}$~~

none

no local extrema

62) $p(x) = \dfrac{x^2-6x+9}{x+2}$

P.N.
$x=-2$
$x=-7$
$x=3$

$p'(x) = \dfrac{(x+2)(2x-6)-(1)(x^2-6x+9)}{(x+2)^2}$

C.V.
$x=-7$
$x=3$

$p'(x) = \dfrac{2x^2-6x+4x-12-x^2+6x-9}{(x+2)^2}$

$p'(x) = \dfrac{x^2+4x-21}{(x+2)^2} = \dfrac{(x+7)(x-3)}{(x+2)^2}$

$f'(x)$: $+$ 0 $-$ N.D. $-$ 0 $+$ at -7, -2, 3

$x=-7$ local max
$x=3$ local min

63) $b(x) = \dfrac{-2x}{(x-3)^5}$

P.N.
$6x+6=0$
$x=\frac{-6}{8}$
$x=-\frac{3}{4}$
$x=3$

$b'(x) = \dfrac{(x-3)^5(-2)-[5(x-3)^4(1)](-2x)}{(x-3)^{10}}$

$b'(x) = \dfrac{(x-3)^4[-2x+6+10x]}{(x-3)^{10}}$

C.V.
$x=-\frac{3}{4}$

$b'(x) = \dfrac{8x+6}{(x-3)^6}$

$f'(x)$: $-$ 0 $+$ N.D. $+$ at $-\frac{3}{4}$, 3

$x=-\frac{3}{4}$ local min

64) $f(x) = x^2(4-x)^2$

P.N. & C.V.
$x=0$
$x=4$
$x=2$

$f(x) = x^2(16-8x+x^2)$

$f(x) = 16x^2-8x^3+x^4$

$f'(x) = 32x-24x^2+4x^3$

$f'(x) = 4x(x^2-6x+8)$

$f'(x) = 4x(x-4)(x-2)$

$f'(x)$: $-$ 0 $+$ 0 $-$ 0 $+$ at 0, 2, 4

$x=0$ local min
$x=2$ local max
$x=4$ local min

65) $f(x) = \sqrt[3]{(2x-1)^2} = (2x-1)^{2/3}$

P.N. & C.V.
$2x-1=0$
$2x=1$
$x=\frac{1}{2}$

$f'(x) = \frac{2}{3}(2x-1)^{-1/3}(2)$

$f'(x) = \dfrac{4}{3\sqrt[3]{2x-1}}$

$f'(x)$: $-$ 0 $+$ at $\frac{1}{2}$

$x=\frac{1}{2}$ local min

66) $g(x) = xe^x$

P.N. & C.V.
$x+1=0$
$x=-1$
$e^x=0$
↳ N.S.

$g'(x) = xe^x + e^x$

$g'(x) = e^x(x+1)$

$-$ 0 $+$ at -1

$x=-1$ local min

67) $j(x) = \ln(x^2-2x+24)$

P.N. & C.V.
$2x-2=0$
$2x=2$
$x=1$
$x^2-2x+24=0$
↳ N.S.

$j'(x) = \dfrac{2x-2}{x^2-2x+24}$

$f'(x)$: $-$ 0 $+$ at 1

$x=1$ local min

68) $g(x) = x^4-2x^3-36x+12$

P.N.
$x=0$ $x=1$

$g'(x) = 4x^3-6x^2-36$

$g''(x) = 12x^2-12x = 12x(x-1)$

$f''(x)$: $+$ 0 $-$ 0 $+$ at 0, 1

CC↑ $(-\infty,0)\cup(1,\infty)$
CC↓ $(0,1)$
I.P. $x=0$, $x=1$

69) $p(x) = \ln(x^2+6x+13)$

P.N.
$x=-3$ & $x=-2$

$p'(x) = \dfrac{2x+6}{x^2+6x+13}$

$p''(x) = \dfrac{(x^2+6x+13)(2)-(2x+6)(2x+6)}{(x^2+6x+13)^2}$

$f''(x)$: $-$ 0 $+$ 0 $-$ at -3, -2

$p''(x) = \dfrac{2x^2+12x+26-4x^2-24x-36}{(x^2+6x+13)^2}$

CC↑: $(-3,-2)$
CC↓: $(-\infty,-3)\cup(-2,\infty)$
I.P. $x=-3$
$x=-2$

$p''(x) = \dfrac{-2x^2-12x-10}{(x^2+6x+13)^2} = \dfrac{-2(x^2+6x+5)}{(x^2+6x+13)^2}$

$p''(x) = \dfrac{-2(x+3)(x+2)}{(x^2+6x+13)^2}$

(70) $h(x) = (x-3)^{1/3}$

P.N. $x = 3$

$h'(x) = \frac{1}{3}(x-3)^{-2/3}$

$h''(x) = \frac{-2}{9}(x-3)^{-5/3}$

$h''(x) = \dfrac{-2}{9(x-3)^{5/3}}$

$f''(x)$: $+$ | 0 | $-$ at 3

CC↑: $(-\infty, 0)$
CC↓: $(0, \infty)$
$x = 3$ I.P.

(71) $g(x) = x^{8/3} - 4x^{5/3}$

$g'(x) = \frac{8}{3}x^{5/3} - \frac{20}{3}x^{2/3}$

$g''(x) = \frac{40}{9}x^{2/3} - \frac{40}{9}x^{-1/3}$

$g''(x) = \dfrac{40x^{2/3}}{9} - \dfrac{40}{9x^{1/3}}$

$g''(x) = \dfrac{40x - 40}{9x^{1/3}}$

P.N.
$40x - 40 = 0$
$40x = 40$
$x = 1$
$x = 0$

$f''(x)$: $+$ | $-$ | $+$ at 0, 1

CC↑ $(-\infty, 0) \cup (1, \infty)$
CC↓ $(0, 1)$
I.P. $x = 0$
$x = 1$

(72) $h(x) = e^{3x} - 9e^{x}$

$h'(x) = 3e^{3x} - 9e^{x}$

$h''(x) = 9e^{3x} - 9e^{x}$

$h''(x) = 9e^{x}(e^{2x} - 1)$

P.N.
$9e^{x} \neq 0$ ↳ N.S
$e^{2x} - 1 = 0$
$e^{2x} = 1$
$2x = \ln 1$
$2x = 0$
$x = 0$

$f''(x)$: $-$ | 0 | $+$ at 0

CC↑ $(0, \infty)$
CC↓ $(-\infty, 0)$
I.P. $x = 0$

(73) $h(x) = -x(x-3)^2 = -x(x^2 - 6x + 9)$

$h(x) = -x^3 + 6x^2 - 9x$

$h'(x) = -3x^2 + 12x - 9$

$h''(x) = -6x + 12$

P.N.
$-6x + 12 = 0$
$12 = 6x$
$2 = x$

$f''(x)$: $+$ | 0 | $-$ at 2

CC↑ $(-\infty, 2)$
CC↓ $(2, \infty)$
I.P. $x = 2$

(74) $j(x) = \dfrac{\ln x}{x^2}$ $x > 0$

$j'(x) = \dfrac{(x^2)(\frac{1}{x}) - (2x)(\ln x)}{x^4}$

$j'(x) = \dfrac{x - 2x\ln x}{x^4} = \dfrac{x(1 - 2\ln x)}{x^4} = \dfrac{1 - 2\ln x}{x^3}$

$j''(x) = \dfrac{(x^3)(-\frac{2}{x}) - (3x^2)(1 - 2\ln x)}{x^6}$

$j''(x) = \dfrac{-2x^2 - 3x^2(1 - 2\ln x)}{x^6} = \dfrac{x^2[-2 - 3 + 6\ln x]}{x^6}$

$j''(x) = \dfrac{-5 + 6\ln x}{x^4}$

PN
$x^4 = 0$
$x \neq 0$
$-5 + 6\ln x = 0$
$6\ln x = 5$
$\ln x = \frac{5}{6}$
$x = e^{5/6}$

$f''(x)$: $-$ | 0 | $+$ at 0, $e^{5/6}$

CC↑ $(e^{5/6}, \infty)$
CC↓ $(0, e^{5/6})$
I.P. $x = e^{5/6}$

(75) $r(x) = (x^3 + 8)^5$

$r'(x) = 5(x^3+8)^4(3x^2) = 15x^2(x^3+8)^4$

$r''(x) = (15x^2)[4(x^3+8)^3(3x^2)] + 30x(x^3+8)^4$

$r''(x) = 180x^4(x^3+8)^3 + 30x(x^3+8)^4$

$r''(x) = (x^3+8)^3[180x^4 + 30x(x^3+8)]$

$r''(x) = (x^3+8)^3(180x^4 + 30x^4 + 240x)$

$r''(x) = (x^3+8)^3(210x^4 + 240x) = (x^3+8)^3[30x(7x^3+8)]$

$r''(x) = 30x(x^3+8)^3(7x^3+8)$

P.N.
$x = 0$
$x^3 + 8 = 0$
$x^3 = -8$
$x = -2$
$7x^3 + 8 = 0$
$7x^3 = -8$
$x^3 = -\frac{8}{7}$
$x = \sqrt[3]{-\frac{8}{7}}$
$x = -1.05$

$-$ | $+$ | $-$ | $+$ at -2, -1.05, 0

CC↑ $(-2, -1.05) \cup (0, \infty)$
CC↓ $(-\infty, -2) \cup (-1.05, 0)$
I.P. $x = -2$
$x = -1.05$
$x = 0$

(26) $h(x) = \frac{x}{x^2+2}$

$$h'(x) = \frac{(x^2+2) - (2x)(x)}{(x^2+2)^2}$$

$$h'(x) = \frac{2-x^2}{(x^2+2)^2} \longrightarrow$$ C.V. $2-x^2=0$, $2=x^2$, $x=\pm\sqrt{2}$

$$h'(x) = \frac{2-x^2}{x^4+4x^2+4}$$

$$h''(x) = \frac{(x^2+2)^2(-2x) - 2(x^2+2)^1(2x)}{(x^2+2)^4}$$

$$h''(\sqrt{2}) = \frac{-32\sqrt{2} - 16\sqrt{2}}{+} = -$$

$x=\sqrt{2}$ local max

$$h''(-\sqrt{2}) = \frac{32\sqrt{2} + 16\sqrt{2}}{+} = +$$

$x=-\sqrt{2}$ local min

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(27) $f(x) = x^3 - 3x^2 - 9x + 1$

$f'(x) = 3x^2 - 6x - 9 = 3(x^2-2x-3) = 3(x-3)(x+1) \longrightarrow$ C.V. $x=3$ $x=-1$

$f''(x) = 6x - 6$

$f''(-1) = -6-6 = -12$ $x=-1$ local max

$f''(3) = 18-6 = 12$ $x=3$ local min

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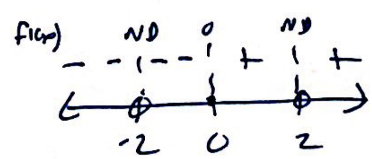
(28) $k(x) = (x^2-4)^{1/3}$

$$k'(x) = \frac{1}{3}(x^2-4)^{-2/3}(2x) = \frac{2x}{3(x^2-4)^{2/3}} \longrightarrow$$

C.V. $2x=0$, $x=0$; $x^2-4=0$, $x^2=4$, $x=\pm 2$

$$k''(x) = \left(\frac{2}{3}x\right)\left(-\frac{2}{3}(x^2-4)^{-5/3}\right)(2x) + \left(\frac{2}{3}\right)(x^2-4)^{-2/3}$$



$k''(2) = $ undef.

$k''(-2) = $ undef

} Test fails use 1st deriv. test $\downarrow$ no extrema

$k''(0) = 0 + \frac{2}{3}(-4)^{-2/3} = +$

$\hookrightarrow$ $x=0$ local min

(79) $b(x) = \frac{\ln x}{x^2} \longrightarrow x > 0$

$b'(x) = \frac{1 - 2\ln x}{x^3}$ } from #74

$b''(x) = \frac{-5 + 6\ln x}{x^4}$

C.V.

$1 - 2\ln x = 0$

$1 = 2\ln x$

$\frac{1}{2} = \ln x$

$e^{1/2} = x$

$b''(e^{1/2}) = \frac{-5 + 6\ln e^{1/2}}{(e^{1/2})^4}$

$b''(e^{1/2}) = \frac{-5 + 3}{e^2} = \frac{-}{+} = -$

$x = e^{1/2}$ local max

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(80) $v(x) = x + 3x^{2/3}$

$v'(x) = 1 + 2x^{-1/3} = 1 + \frac{2}{x^{1/3}} = \frac{x^{1/3} + 2}{x^{1/3}}$ $\longrightarrow$ C.V.

$x^{1/3} + 2 = 0$ $\quad x = 0$

$x^{1/3} = -2$

$x = -8$

$v''(x) = -\frac{2}{3}x^{-4/3} = -\frac{2}{3x^{4/3}}$

f'(x): + | − | + ; at -8: 0, at 0: ND

$v''(-8) = \frac{-2}{3(-8)^{4/3}} = -$ $\quad$ $x = -8$ local max

$v''(0) =$ undefined $\rightarrow$ 2nd deriv. test fails : use 1st deriv. test

$x = 0$ local min

(81) $m(x) = x + e^{-3x}$

$m'(x) = 1 - 3e^{-3x}$ $\longrightarrow$ C.V.

$1 - 3e^{-3x} = 0$

$1 = 3e^{-3x}$

$\frac{1}{3} = e^{-3x}$

$\ln \frac{1}{3} = -3x$

$\frac{\ln \frac{1}{3}}{-3} = x$

$m''(x) = 9e^{-3x} = \frac{9}{e^{3x}}$

$m''\left(\frac{\ln \frac{1}{3}}{-3}\right) = \frac{9}{e^{3\left(\frac{\ln 1/3}{-3}\right)}}$

$m''\left(\frac{\ln 1/3}{-3}\right) = \frac{9}{e^{-\ln 1/3}} = \frac{9}{e^{\ln 3}} = \frac{9}{3} = 3 = +$

$x = \frac{\ln 1/3}{-3}$ local min

(82) $f(x) = (x+4)(x-2)^2$

$f(x) = (x+4)(x^2-4x+4)$

$f(x) = x^3-4x^2+4x+4x^2-16x+16$

$f(x) = x^3-12x+16$

$f'(x) = 3x^2-12 \longrightarrow$ C.V. $3x^2-12=0$

$3(x^2-4)=0$

$f''(x) = 6x$

$3(x+2)(x-2)=0$

$x=-2 \quad x=2$

$f''(-2) = -12$

$x=-2$ local max

$f''(2) = 12$

$x=2$ local min

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(83) $f(x) = \dfrac{x^2+4}{x}$

$f(x) = x+4x^{-1}$

$f'(x) = 1-4x^{-2} = 1-\dfrac{4}{x^2} = \dfrac{x^2-4}{x^2}$

C.V. $x^2-4=0$

$x=\pm 2$

not $x=0 \rightarrow$ not in Domain of $f(x)$

$f''(x) = 8x^{-3} = \dfrac{8}{x^3}$

$f''(2) = \dfrac{8}{8} = 1$

$x=2$ local min

$f''(-2) = \dfrac{8}{-8} = -1$

$x=-2$ local max

(84) $\ln y + x^2 y = 5$

$\dfrac{1}{y}\left(\dfrac{dy}{dx}\right) + x^2\left(\dfrac{dy}{dx}\right) + 2xy = 0$

$\dfrac{dy}{dx}\left(\dfrac{1}{y}+x^2\right) = -2xy$

$\dfrac{dy}{dx} = \dfrac{-2xy}{\frac{1}{y}+x^2}$ or $\dfrac{dy}{dx} = \dfrac{-2xy^2}{1+x^2y}$

(85) $\frac{x}{y} + 5 = e^x$

$$\frac{y - \left(\frac{dy}{dx}\right)(x)}{y^2} = e^x$$

$$y - \left(\frac{dy}{dx}\right)(x) = e^x y^2$$

$$-x\left(\frac{dy}{dx}\right) = y^2e^x - y$$

$$\frac{dy}{dx} = \frac{y^2e^x - y}{-x} \text{ or } \frac{y - y^2e^x}{x}$$

(86) $(2x+y^2)^4 = \ln y + 2x$

$$4(2x+y^2)^3\left(2 + 2y\left(\frac{dy}{dx}\right)\right) = \frac{1}{y}\left(\frac{dy}{dx}\right) + 2$$

$$8(2x+y^2)^3 + 8y(2x+y^2)^3\frac{dy}{dx} = \frac{1}{y}\left(\frac{dy}{dx}\right) + 2$$

$$\frac{dy}{dx}\left[8y(2x+y^2)^3 - \frac{1}{y}\right] = 2 - 8(2x+y^2)^3$$

$$\frac{dy}{dx} = \frac{2 - 8(2x+y^2)^3}{8y(2x+y^2)^3 - \frac{1}{y}}$$

(87) $(xy)^2 + 7 = e^{-x}$

$$2(xy)^1\left(x\left(\frac{dy}{dx}\right) + y\right) = -e^{-x}$$

$$2x^2y\left(\frac{dy}{dx}\right) + 2xy^2 = -e^{-x}$$

$$2x^2y\left(\frac{dy}{dx}\right) = -e^{-x} - 2xy^2$$

$$\frac{dy}{dx} = \frac{-e^{-x} - 2xy^2}{2x^2y}$$

(88) $e^{x^2+y^2} = x\ln y + 7$

$$\left(e^{x^2+y^2}\right)\left(2x + 2y\frac{dy}{dx}\right) = (x)\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) + \ln y$$

$$2xe^{x^2+y^2} + 2ye^{x^2+y^2}\left(\frac{dy}{dx}\right) = \frac{x}{y}\frac{dy}{dx} + \ln y$$

$$\frac{dy}{dx}\left[2ye^{x^2+y^2} - \frac{x}{y}\right] = \ln y - 2xe^{x^2+y^2}$$

$$\frac{dy}{dx} = \frac{\ln y - 2xe^{x^2+y^2}}{2ye^{x^2+y^2} - \frac{x}{y}}$$

(89) $\ln(\ln(y)) + \ln e^x = e^{-y}$

$$\ln(\ln y) + x = e^{-y}$$

$$\frac{\frac{1}{y}\left(\frac{dy}{dx}\right)}{\ln y} + 1 = -e^{-y}\frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} + \ln y = -(\ln y)(e^{-y})\left(\frac{dy}{dx}\right)$$

$$\frac{dy}{dx}\left[\frac{1}{y} + (\ln y)(e^{-y})\right] = -\ln y$$

$$\frac{dy}{dx} = \frac{-\ln y}{\frac{1}{y} + (\ln y)(e^{-y})}$$

(90) $x(2x^2+y^2)^3 = e^y$

$$x\left[3(2x^2+y^2)^2\left(4x+2y\left(\frac{dy}{dx}\right)\right)\right] + (2x^2+y^2)^3 = e^y \frac{dy}{dx}$$

$$12x^2(2x^2+y^2)^2 + 6xy(2x^2+y^2)^2\left(\frac{dy}{dx}\right) + (2x^2+y^2)^3 = e^y \frac{dy}{dx}$$

$$12x^2(2x^2+y^2)^2 + (2x^2+y^2)^3 = \frac{dy}{dx}\left[e^y - 6xy(2x^2+y^2)^2\right]$$

$$\frac{12x^2(2x^2+y^2)^2 + (2x^2+y^2)^3}{e^y - 6xy(2x^2+y^2)^2} = \frac{dy}{dx}$$

(91) $\frac{1}{(x^2y)^3} - e^{-x} = 7$

$$\frac{1}{x^6y^3} - e^{-x} = 7$$

$$x^{-6}y^{-3} - e^{-x} = 7$$

$$(x^{-6})(-3y^{-4})\left(\frac{dy}{dx}\right) + (-6x^{-7})(y^{-3}) + e^{-x} = 0$$

$$-3x^{-6}y^{-4}\left(\frac{dy}{dx}\right) = 6x^{-7}y^{-3} - e^{-x}$$

$$\frac{dy}{dx} = \frac{6x^{-7}y^{-3} - e^{-x}}{-3x^{-6}y^{-4}}$$

(92) $x^2e^x + ye^y + e^x = 0$

$x^2(e^x) + 2x(e^x) + y\, e^y\left(\frac{dy}{dx}\right) + \frac{dy}{dx}(e^y) + e^x = 0$

$ye^y\left(\frac{dy}{dx}\right) + e^y\left(\frac{dy}{dx}\right) = -x^2e^x - 2xe^x - e^x$

$\frac{dy}{dx}\left(ye^y + e^y\right) = -x^2e^x - 2xe^x - e^x$

$$\frac{dy}{dx} = \frac{-e^x(x^2+2x+1)}{ye^y + e^y} = \frac{-e^x(x+1)(x+1)}{e^y(y+1)}$$

$\frac{dy}{dx} = 0$ when top $= 0$ $\quad x+1=0$

$x = -1$

(93) $y^3 + xy^2 + 1 = x + 2y^2$ at $x=2$

Find y $\quad y^3 + \cancel{2y^2} + 1 = \cancel{2} + \cancel{2y^2}$

$y^3 = 1$

$y = 1$ $\quad (2,1)$

Find $\frac{dy}{dx}$ $\quad 3y^2\frac{dy}{dx} + 2xy\frac{dy}{dx} + y^2 = 1 + 4y\frac{dy}{dx}$

$\frac{dy}{dx}(3y^2 + 2xy - 4y) = 1 - y^2$

$$\frac{dy}{dx} = \frac{1-y^2}{3y^2 - 2xy - 4y}$$

Find Slope

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{1-(1)^2}{3(1)^2 - 2(2)(1) - 4(1)} = 0$$

$y = mx + b$

$1 = 0(2) + b$

$1 = b$

$y = 0x + 1$

$y = 1$

(94) $y^2 - 3xy = 10$ at $x=1$

Find y

$y^2 - 3y - 10 = 0$

$(y-5)(y+2)=0$

$y=5 \quad y=-2$

$(1,5) \quad (1,-2)$

Find $\frac{dy}{dx}$

$2y\frac{dy}{dx} - 3x\frac{dy}{dx} - 3y = 0$

$\frac{dy}{dx} = \frac{3y}{2y-3x}$

$(1,5)$

$\frac{dy}{dx} = \frac{3(5)}{2(5)-3}$

$\frac{dy}{dx} = \frac{15}{7}$

$y = mx+b$

$5 = \frac{15}{7}(1)+b$

$\frac{20}{7} = b$

$y = \frac{15}{7}x + \frac{20}{7}$

$(1,-2)$

$\frac{dy}{dx} = \frac{3(-2)}{2(-2)-3(1)}$

$\frac{dy}{dx} = \frac{-6}{-7} = \frac{6}{7}$

$y = mx+b$

$-2 = \frac{6}{7}(1)+b$

$-\frac{20}{7} = b$

$y = \frac{6}{7}x - \frac{20}{7}$

(95) $x^2 - 3xy + y^2 = 5 \qquad x=4 \quad y=1 \quad \frac{dy}{dt}=2$

$2x\left(\frac{dx}{dt}\right) - \left[3x\left(\frac{dy}{dt}\right) + 3\left(\frac{dx}{dt}\right)(y)\right] + 2y\left(\frac{dy}{dt}\right) = 0$

$2(4)\left(\frac{dx}{dt}\right) - 3(4)(2) - 3\left(\frac{dx}{dt}\right)(1) + 2(1)(2) = 0$

$8\left(\frac{dx}{dt}\right) - 24 - 3\left(\frac{dx}{dt}\right) + 4 = 0$

$5\left(\frac{dx}{dt}\right) = 20$

$\frac{dx}{dt} = 4$

(96) $V = \frac{4}{3}\pi r^3 \qquad \frac{dv}{dt} = ? \qquad r=2 \quad \frac{dr}{dt} = 1$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dv}{dt} = 4\pi(2)^2(1)$

$\frac{dv}{dt} = 16\pi \ mm^3/min$

(97) $V = \frac{1}{3}\pi r^2 h \qquad \frac{dv}{dt} = 30 \qquad \frac{dh}{dt} = ? \quad h = 10$

$h = \frac{1}{2}r$

$2h = r$

$v = \frac{1}{3}\pi(2h)^2 h$

$v = \frac{1}{3}\pi(4h^2)h$

$v = \frac{4}{3}\pi h^3$

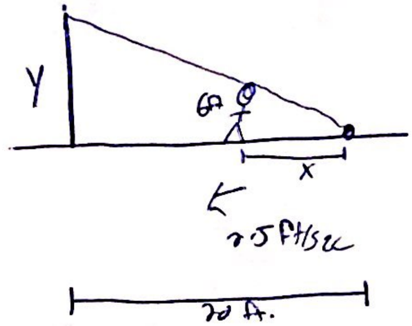
$\frac{dv}{dt} = 4\pi h^2\left(\frac{dh}{dt}\right)$

$30 = 4\pi(10)^2\left(\frac{dh}{dt}\right)$

$\frac{30}{400\pi} = \frac{dh}{dt}$

$\frac{3}{40\pi} \ ft/min = \frac{dh}{dt}$

(98)



$\frac{dx}{dt} = 2.5$

$\frac{dy}{dt} = ?$

$x = 12$

$\frac{y}{6} = \frac{20}{x}$

$\frac{1}{6}y = 20x^{-1}$

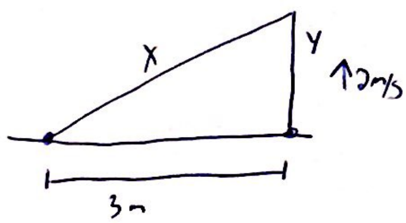
$\frac{1}{6}\left(\frac{dy}{dt}\right) = -20x^{-2}\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{-120}{x^2}\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{-120}{(12)^2}(2.5) = \frac{-10}{12}(2.5)$

$\frac{dy}{dt} = \frac{-25}{12}$ ft/sec

(99)



$\frac{dy}{dt} = 2$

$y = 4$

$\frac{dx}{dt} = ?$

Find x

$3^2 + y^2 = x^2$

$9 + 16 = x^2$

$25 = x^2$

$x = 5$

$3^2 + y^2 = x^2$

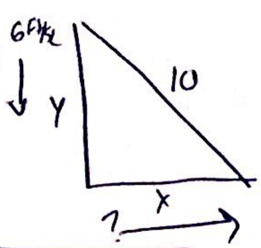
$\cancel{2}y\left(\frac{dy}{dt}\right) = \cancel{2}x\left(\frac{dx}{dt}\right)$

$\frac{(4)(2)}{5} = \frac{dx}{dt}$

$\frac{8}{5} = \frac{dx}{dt}$

8/5 m/sec

(100)



$\frac{dy}{dt} = -6$

$y = 8$

$\frac{dx}{dt} = ?$

$x^2 + y^2 = 10^2$	$x^2 = 36$
$x^2 + 64 = 100$	$x = 6$

$\longrightarrow x = 6$

$x^2 + y^2 = 10^2$

$\cancel{2}x\left(\frac{dx}{dt}\right) + \cancel{2}y\left(\frac{dy}{dt}\right) = 0$

$\frac{dx}{dt} = \frac{-y\left(\frac{dy}{dt}\right)}{x}$

$\frac{dx}{dt} = \frac{-8(-6)}{6}$

$\frac{dx}{dt} = 8$ ft/sec